Instructions: *Do any five of the six problems below. Turn in ONLY the five problems you want graded. Write your name and the problem number clearly at the top of each page you turn in for grading. You have three hours.* Good luck!

1. Find all nonisomorphic abelian groups of order 1000. For each group, list the elementary divisors AND the invariant factors.

2. Let p be a prime number and let G be a group of order p^2. Prove that G is abelian.

3. Prove Cauchy's theorem: If the prime number p divides the order of a finite group G, then G contains an element of order p.

4. This problem has four parts. Let I and J be ideals in a commutative ring R with 1.

   a) Give the definition of $I + J$.

   b) Give the definition of IJ.

   c) Define what it means for the ideals I and J to be comaximal.

   d) Let the ideals I and J be comaximal. Prove that $IJ = I \cap J$.

5. Let R be a commutative ring with 1, and let M be an R-module. Define what it means for M to be R-cyclic. Then prove that M is R-cyclic if and only if $M \cong R/I$ for some ideal I in R.

6. Let $\mathbb{Q}$ denote the field of rational numbers and let $T : \mathbb{Q}^5 \longrightarrow \mathbb{Q}^5$ be a linear transformation with minimal polynomial
$$(X^2 - 4)(X^2 + 2)$$
   Find all choices for T up to similarity. For each T list the invariant factors AND the elementary divisors of the corresponding $\mathbb{Q}[X]-$module $\mathbb{Q}^5$.