Instructions: *Work two problems from each section for a total of four problems.* Be sure to write the number for each problem with your work, and write your name clearly at the top of each page you turn in for grading. You have three hours.

A Point Set Topology

A1. (a) Show that if $f : X \to Y$ is a continuous bijection from a compact space X to a Hausdorff space Y, then f is a homeomorphism.

(b) Give an example of topological spaces X and Y and a continuous bijection $f : X \to Y$ that is **not** a homeomorphism.

A2. Give an example of a connected space that is not path-connected, and prove that it has the stated properties.

A3. Let $f : X \to Y$ be a quotient map of topological spaces, such that Y is connected and each set $f^{-1}(y)$, $y \in Y$, is a connected subspace of X. Show that X is connected.

A4. (a) Show that a metric space is normal.

(b) Show that a compact Hausdorff space is normal.

B Homotopy

B1. Let X be a path connected space. Prove that $\pi_1(X, x)$ is isomorphic to $\pi_1(X, y)$ for all points $x, y \in X$.

B2. (a) Precisely state the Seifert-Van Kampen theorem.

(b) Give an example of a space whose fundamental group is a cyclic group of order six.

(c) Prove that the fundamental group of the space you constructed in (b) is cyclic.

B3. Prove (using homotopy theory) that every continuous map $f : D^2 \to D^2$ has a fixed point.

B4. Let $p : (E, e_0) \to (X, x_0)$ where p is a covering map (X and E are path connected, locally path connected). Let $e \in p^{-1}(x_0)$. Prove that there exists a homeomorphism $f : E \to E$ in the group of covering transformations of E such that $f(e_0) = e$ if and only if $p_*\pi_1(E, e_0) = p_*\pi_1(E, e)$.