Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Let $f : S^1 \to \mathbb{R}$ be continuous, where S^1 is the unit circle in $\mathbb{R}^2$.

i. Show that there is a point $x \in S^1$ such that $f(x) = f(-x)$

ii. Show that f is not surjective.

A2. Prove that the product of finitely many connected spaces is connected.

A3. Let $f : X \to Y$ be a continuous function between topological spaces X and Y, and let A be a subset of X.

i. If A is compact, prove that $f(A)$ is compact.

ii. If A is connected, prove that $f(A)$ is connected.

B. **Homotopy** (2 problems)

B1. Let $p : E \to B$ be a covering map with connected base space B, and let x and y be two points in B. Show that the sets $p^{-1}(x)$ and $p^{-1}(y)$ have the same cardinality.

B2. Prove that $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$ for $n > 2$.

B3. Prove the Brouwer fixed point theorem in dimension two: Every continuous map $f : D^2 \longrightarrow D^2$ has a fixed point.

C. **Mixed** (1 problem)

C1. Show that the following three conditions on a topological space X are equivalent:

i. Every continuous map $S^1 \longrightarrow X$ is null homotopic.

ii. Every continuous map $S^1 \longrightarrow X$ extends to a continuous map $D^2 \longrightarrow X$.

iii. The fundamental group $\pi_1(X, x_0)$ is trivial for all $x_0 \in X$.

C2. Suppose X and Y are topological spaces, and let $x_0 \in X$ and $y_0 \in Y$. Prove that $\pi_1(X \times Y, x_0 \times y_0) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0)$.