Topology **Comprehensive/Qualifying Exam** **August, 2015**

Instructions: *You must work two problems from Section A, two from Section B, and one additional problem from either of the two sections (for a total of *five* problems).* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

Section A:

A1 Let A be a subspace of topological space X. A continuous map $r : X \to A$ is said to be a *retraction* if $r(a) = a$ for ever $a \in A$.

(a) Show that if $r : X \to A$ is a retraction, then the induced map $r_* : \pi_1(X, x_0) \to \pi_1(A, a_0)$ is surjective. Conclude that there does not exist a retraction $r : D^2 \to S^1$.

(b) Prove that every continuous map : $D^2 \to D^2$ has a fixed point.

A2 Prove that the retract of a contractible space is contractible.

A3 Recall that a (non-empty) subset X of $\mathbb{R}^n$ is *convex* if the line segment between any two points in X is contained in X. If X is convex, show that X is simply connected.

A4 Give an example of a connected space that is not path-connected, and prove that it has the stated property.

Section B:

B1 Let $X = S^1 \vee S^1$ be the wedge of two circles. Give examples of normal and non-normal 3–fold covers of X (justify your examples). Does X have a non-normal 2-fold covering (justify your)?

B2 Let X be a path-connected and locally path-connected space with finite fundamental group. Show that any map $f : X \to S^1$ is null-homotopic.

B3 Let $p : E \to X$ be the covering map indicated in Figure 1 below. Determine the group of covering transformations. Is the covering normal?

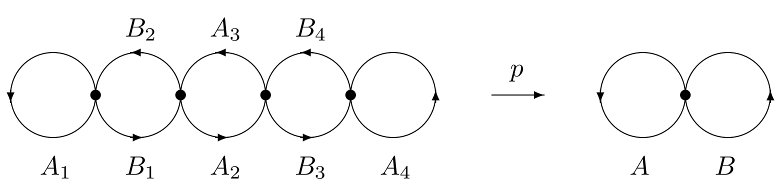

Figure 1: A covering of the wedge of two circles.

B4 Give an example of a space whose fundamental group is a cyclic group of order six. Prove that your example does indeed have this fundamental group.
