Instructions: *You must work two problems from Section A, two from Section B, and one additional problem from either of the two sections (for a total of *five* problems).* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

Section A:

A1 Let Σ_g denote the closed surface of genus g. Show that $\Sigma_n \cong \Sigma_m$ if and only if $n = m$.

A2 Let $p : X \to Y$ be a quotient map, and $f : X \to Z$ a continuous function such that $f(x_1) = f(x_2)$ whenever $p(x_1) = p(x_2)$. Show that there is a unique function $g : Y \to Z$ such that $g \circ p = f$, and that g is continuous.

A3 Let A be a subspace of topological space X. A continuous map $r : X \to A$ is said to be a *retraction* if $r(a) = a$ for ever $a \in A$.

(a) Show that if $r : X \to A$ is a retraction, then the induced map $r_* : \pi_1(X, x_0) \to \pi_1(A, a_0)$ is surjective. Conclude that there does not exist a retraction $r : D^2 \to S^1$.

(b) Prove that every continuous map : $D^2 \to D^2$ has a fixed point.

A4 Show that if B is simply-connected, then any covering map $p : E \to B$ for which E is path-connected is a homeomorphism.

Section B:

B1 Let $p : E \to B$ be a covering map with connected base space B, and let x and y be two points in B. Show that the sets $p^{-1}(x)$ and $p^{-1}(y)$ have the same cardinality.

B2 Let $X = S^1 \vee S^1$ be the wedge of two circles. Give examples of normal and non-normal 4–fold covers of X (justify your examples). Does X have a non-normal 2–fold covering (justify your)?

B3 Find all connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.

B4 Let Σ be a surface of genus $g \geq 1$ and S^n a sphere of dimension $n \geq 2$. Show that any continuous map

$$f : S^n \to \Sigma$$

is homotopically trivial.