Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Let $f : X \to Y$ be a continuous function between topological spaces X and Y, and let A be a subset of X.

i. If A is compact, prove that $f(A)$ is compact.

ii. If A is connected, prove that $f(A)$ is connected.

A2. Prove that the product of finitely many connected spaces is connected.

A3. i. Let X be a Hausdorff space and A be a compact subset of X. Prove that A is closed.

ii. Let $f : X \longrightarrow Y$ be a continuous map from a compact space X to a Hausdorff space Y . Prove that f is a closed map.

B. **Homotopy** (2 problems)

B1. State the Seifert-Van Kampen Theorem. Use this theorem to show that the n-sphere S^n is simply connected for $n \geq 2$.

B2. Prove that $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$ for $n \neq 2$.

B3. Prove the Brouwer fixed point theorem in dimensions one and two: Every continuous map $f : D^n \longrightarrow D^n$, $n = 1, 2$, has a fixed point.

C. **Mixed** (1 problem)

C1. Show that the following three conditions on a topological space X are equivalent:

i. Every continuous map $S^1 \longrightarrow X$ is null homotopic.

ii. Every continuous map $S^1 \longrightarrow X$ extends to a continuous map $D^2 \longrightarrow X$.

iii. The fundamental group $\pi_1(X, x_0)$ is trivial for all $x_0 \in X$.

C2. The real projective space RP^n is the quotient space of S^n obtained by identifying antipodal points.

i. What is RP^1?

ii. Determine the fundamental group of RP^n for all $n \geq 1$.
