Core I Topology Exam

January, 2004

Instructions: Do problems 1* and 2* and any two others for a total of four problems. Put your name on the top right corner of each sheet, and write on only one side of each sheet. Copy each problem before solving it. Turn in ONLY the four problems you want graded. Think carefully about your write-up; clarity, logic, and completeness of your solutions will be considered when the problems are graded. GOOD LUCK!

1*.

a. Let X be a Hausdorff space and A be a compact subset of X. Prove that A is closed.

b. Let $f : X \longrightarrow Y$ be a continuous map from a compact space X to a Hausdorff space Y. Prove that f is a closed map.

2*. Let (X, d) be a metric space. Prove or disprove: There is a **bounded metric** δ which gives the same topology on X as the topology given by the metric d.

3. Let X, Y be topological spaces, $X = A \cup B$ where A, B are closed subsets of X, and $f : X \longrightarrow Y$ be a function such that the restrictions $f\big|_A$ and $f\big|_B$ are continuous with respect to the relative topologies on A, B, respectively. Prove that f is continuous.

4. Let (X, d) be a complete metric space (a metric space in which Cauchy sequences converge), and let $f : X \longrightarrow X$ be a function for which $d(f(x), f(y)) \leq \frac{1}{2} d(x, y)$ for all $x, y \in X$. Prove that f has exactly one fixed point.

5. Prove or disprove: Every finite Hausdorff space is metrizeable.