Instructions: *Work two problems from Section A, two problems from Section B, and one problem from Section C, for a total of five problems.* Be sure to write the number of each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

A. **Point Set Topology** (2 problems)

A1. Let A be a subspace of a compact Hausdorff space X. Prove that A is compact if and only if A is closed in X.

A2. Let $f\colon S^1 \to \mathbb{R}$ be continuous, where S^1 is the unit circle in $\mathbb{R}^2$.

i. Show that there is a point $z \in S^1$ such that $f(z) = f(-z)$.

ii. Show that f is not surjective.

A3. Let $f\colon X \to Y$ be a quotient map of topological spaces such that Y is connected and for each $y \in Y$, the set $f^{-1}(y)$ is a connected subspace of X. Show that X is connected.

B. **Homotopy** (2 problems)

B1. i. Show that a retract of a contractible space is contractible.

ii. Show that the unit circle S^1 is not a retract of the unit disk D^2.

B2. Let X be a path-connected, locally path-connected space, and supposed that the fundamental group of X is finite. Prove that any continuous map $f\colon X \to S^1$ is nullhomotopic.

B3. i. State precisely the Seifert-van Kampen Theorem.

ii. Prove that there does not exist an open cover $\{U, V\}$ of the real projective plane $\mathbb{R}P^2$ such that U and V are both contractible and $U \cap V$ is connected.

C. **Mixed** (1 problem)

C1. Prove that no two of the following three spaces are homeomorphic:

i. the punctured plane $\mathbb{R}^2 - \{(0,0)\}$

ii. the unit circle S^1

iii. the unit sphere S^2

C2. Let X be the sphere S^2 with three points removed, and let $Y = S^1 \vee S^1$ be the figure eight.

i. Is X homotopy equivalent to Y? Justify your answer.

ii. Is X homeomorphic to Y? Justify your answer.