**Instructions**: *Do* **any four** *of the following six problems. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have two and a half hours.* Good luck!

---

1. Prove that a product of (finitely or infinitely many) connected spaces is connected.

2. Prove that a function $f\colon X \to Y$ between metric spaces is continuous if and only if $f(x_n) \to f(x)$ whenever $x_n \to x$ is a convergent sequence in $X$.

3. Let $(X_\alpha)_{\alpha\in J}$ be a family of topological spaces indexed by an arbitrary set $J$, and let $A_\alpha$ be a subset of $X_\alpha$ for $\alpha \in J$. Show that $\overline{\prod_{\alpha\in J} A_\alpha} = \prod_{\alpha\in J} \overline{A_\alpha}$.

4. Let $p\colon X \to Y$ be a continuous surjection. Show that if $X$ is compact and $Y$ is Hausdorff then $p$ is a quotient map.

5. Let $f\colon S^1 \to \mathbb{R}$ be continuous, where $S^1$ is the unit circle in $\mathbb{R}^2$.

   (a) Show that there is a point $z \in S^1$ such that $f(z) = f(-z)$.

   (b) Show that $f$ is not surjective.

6. Let $X$ be a $T_0$ space. (That is, for any two distinct points of $X$, there is an open set containing exactly one of them.) Suppose that, for any $x \in X$ and closed subset $A$ of $X$ not containing $x$, there are disjoint open sets $U \ni x$ and $V \supseteq A$. Prove that $X$ is Hausdorff.