Core 1 Topology Exam

January 2005

Do **any four** of the following problems. Indicate clearly which problems you have chosen. You have 2 and 1/2 hours to complete this test. Good luck!

1. Let C be a subset of a topological space X.

(a) Prove that if C is connected, then the closure of C is connected.

(b) Give an example where C is connected but the interior of C is not connected. (You may wish to take $X = R^2$.)

2. Prove that a connected normal space with at least two points is uncountable. [Hint: use Urysohn's Lemma.]

3. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be topologies on a set X with $\mathcal{T}_1 \subseteq \mathcal{T}_2$. Show that if $(X, \mathcal{T}_1)$ is Hausdorff and $(X, \mathcal{T}_2)$ is compact then $\mathcal{T}_1 = \mathcal{T}_2$.

4. Let $p : X \to Y$ be a quotient map, and $f : X \to Z$ a continuous function such that $f(x_1) = f(x_2)$ whenever $p(x_1) = p(x_2)$. Show that there is a unique function $g : Y \to Z$ such that $g \circ p = f$, and that g is continuous.

5. (a) Show that a path-connected space is connected.

(b) Show that a connected, locally path-connected space is path-connected.

6. Show that a finite Hausdorff space is metrizable.