Instructions: *Do* **any four** *of the following six problems. Be sure to write the number for each problem you work out and write your name clearly at the top of each page you turn in for grading. You have two and a half hours.* Good luck!

1. Prove that a compact Hausdorff space is normal.
2. Let X be a metric space. Prove that X is separable if and only if it is second countable.
3. Prove that a countable product of metrizable spaces is metrizable.
4. Let $p\colon X \to Y$ be a closed, continuous surjection. Prove that if Y is compact and $p^{-1}(y)$ is compact for every $y \in Y$, then X is compact. [Hint: if U is an open set containing $p^{-1}(y)$, there is a neighborhood V of y such that $p^{-1}(V) \subseteq U$.]
5. Let $f\colon [-1,1] \to [-1,1]$ be a continuous function. Prove that there is a point x_0 of $[-1,1]$ such that $f(x_0) = x_0$. [Hint: consider $g(x) = (x - f(x))/|x - f(x)|$.]
6. Give an example of a connected space that is not path-connected, and prove that it has the stated properties.