Instructions: *You must work two problems from Section A, two from Section B, and one additional problem from either of the two sections (for a total of *five* problems).* Be sure to write the number for each problem you work out, and write your name clearly at the top of each page you turn in for grading. You have three hours. Good luck!

Section A:

A1 Prove that $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$ for any $n \neq 2$.

A2 Let $f : X \to Y$ be a continuous function between topological spaces X and Y, and let A be a subset of X.

(a) If A is compact, prove that f(A) is compact.

(b) If A is connected, prove that f(A) is connected.

A3 Let A be a subspace of topological space X. A continuous map $r : X \to A$ is said to be a *retraction* if $r(a) = a$ for ever $a \in A$.

(a) Show that if $r : X \to A$ is a retraction, then the induced map $r_* : \pi_1(X, x_0) \to \pi_1(A, a_0)$ is surjective. Conclude that there does not exist a retraction $r : D^2 \to S^1$.

(b) Prove that every continuous map : $D^2 \to D^2$ has a fixed point.

A4 Prove that the retract of a contractible space is contractible.

Section B:

B1 Let $X = S^1 \vee S^1$ be the wedge of two circles. Give examples of normal and non-normal 3–fold covers of X (justify your examples). Does X have a non-normal 2-fold covering (justify your)?

B2 Let X be a path-connected and locally path-connected space with finite fundamental group. Show that any map $f : X \to S^1$ is null-homotopic.

B3 Let X and Y be topological spaces, and let $x_0 \in X$ and $y_0 \in Y$. Show that

$$\pi_1(X \times Y, x_0 \times y_0) = \pi_1(X, x_0) \times \pi_1(Y, y_0).$$

B4 Construct a space X that has fundamental group $\pi_1(X, x_0) \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$. Justify your answer.