GUI for Electromagnetic Waves in Layered Media

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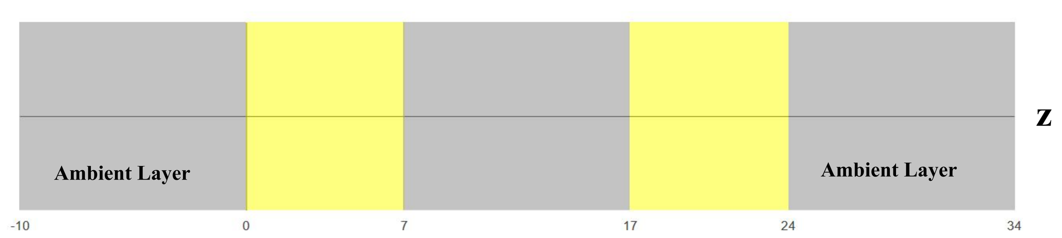
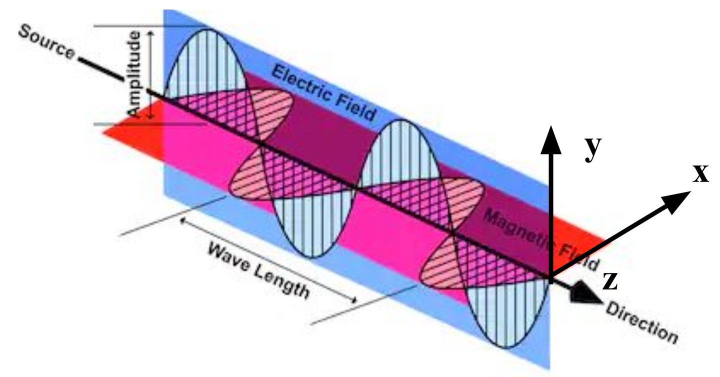
Abstract

The goal is to make available to the community a versatile, online application for the computation of electromagnetic fields in media with any number of layers having arbitrary electric and magnetic tensors. The objectives are:

(1) to allow scientists to explore phenomena of scattering, guided modes, and resonance in the most general EM layered media

(2) to provide a pedagogical tool for students and professionals to learn EM in layered media.





Harmonic Fields in Layered Media

Layered Structures: We can create a structure made out of any number of layers. The medium in each layer can be represented by μ and ε matrices.

$$\varepsilon = \varepsilon^{(1)} + i\varepsilon^{(2)}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{11}{}^{(1)} & \varepsilon_{12}{}^{(1)} & \varepsilon_{13}{}^{(1)} \\ \varepsilon_{12}{}^{(1)} & \varepsilon_{22}{}^{(1)} & \varepsilon_{23}{}^{(1)} \\ \varepsilon_{13}{}^{(1)} & \varepsilon_{23}{}^{(1)} & \varepsilon_{33}{}^{(1)} \end{bmatrix} + i \begin{bmatrix} \varepsilon_{11}{}^{(2)} & \varepsilon_{12}{}^{(2)} & \varepsilon_{13}{}^{(2)} \\ \varepsilon_{12}{}^{(2)} & \varepsilon_{22}{}^{(2)} & \varepsilon_{23}{}^{(2)} \\ \varepsilon_{13}{}^{(2)} & \varepsilon_{23}{}^{(2)} & \varepsilon_{33}{}^{(2)} \end{bmatrix}$$

$\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are Hermitian Matrices, that is that the matrix is equal to its own conjugate transpose. The same is true for μ.

$$\mu = \mu^{(1)} + i\mu^{(2)}$$

μ is a magnetic tensor and ε is an electric tensor.

Harmonic Field: Each layer will have an electromagnetic field, denoted by E(x,y,z,t) for the electric component and H(x,y,z,t) for the magnetic component. A harmonic field results when there is oscillation in the x, y, and t directions. This is controlled by the wave vector $<k_1,k_2>$ parallel to the layers, and the frequency ω like so

$$E = E(z)e^{i(k_1 x + k_2 y - \omega t)} \qquad H = H(z)e^{i(k_1 x + k_2 y - \omega t)}$$

Scattering Problem: Using the structure built above, we can simulate physical experiments. The experiment consists of a wave source located in each of the ambient layers. The angle of incidence of these fields are related to the wave vector and the frequency is controlled by ω . The resultant total electromagnetic field will be computed and displayed.

Mathematics of Waves in Layered Media

Given an n-layered structure, values of ε and μ for each layer, and ω and $<k_1,k_2>$ values, the Maxwell partial differential equations for E(x,y,z,t) and H(x,y,z,t) reduce to a 4x4 system of ordinary differential equations for the tangential components of E(z) and H(z). In each layer, the system is

$$\frac{d}{dz}F(z) = i\kappa \begin{bmatrix} -\frac{\varepsilon_{31}\kappa_1}{\varepsilon_{33}} - \frac{\mu_{23}\kappa_2}{\mu_{33}} & (-\frac{\varepsilon_{32}}{\varepsilon_{33}} + \frac{\mu_{23}}{\mu_{33}})\kappa_1 & \mu_{21} + \frac{\kappa_1\kappa_2}{\varepsilon_{33}} - \frac{\mu_{23}\mu_{31}}{\mu_{33}} & \mu_{23} - \frac{\kappa_1^2}{\varepsilon_{33}} - \frac{\mu_{23}\mu_{32}}{\mu_{33}} \\ (-\frac{\varepsilon_{31}}{\varepsilon_{33}} + \frac{\mu_{13}\kappa_2}{\mu_{33}}) & -\frac{\varepsilon_{32}\kappa_2}{\varepsilon_{33}} - \frac{\mu_{13}\kappa_1}{\mu_{33}} & -\mu_{11} + \frac{\kappa_2^2}{\varepsilon_{33}} + \frac{\mu_{13}\mu_{31}}{\mu_{33}} & -\mu_{12} - \frac{\kappa_1\kappa_2}{\varepsilon_{33}} + \frac{\mu_{13}\mu_{32}}{\mu_{33}} \\ -\varepsilon_{21} - \frac{\kappa_1\kappa_2}{\mu_{33}} + \frac{\varepsilon_{23}\varepsilon_{31}}{\varepsilon_{33}} & -\varepsilon_{22} + \frac{\kappa_1^2}{\mu_{33}} + \frac{\varepsilon_{23}\varepsilon_{32}}{\varepsilon_{33}} & -\frac{\mu_{31}\kappa_1}{\mu_{33}} - \frac{\varepsilon_{23}\kappa_2}{\varepsilon_{33}} & (-\frac{\mu_{32}}{\mu_{33}} + \frac{\varepsilon_{23}}{\varepsilon_{33}})\kappa_1 \\ \varepsilon_{11} - \frac{\varepsilon_{13}\varepsilon_{31}}{\varepsilon_{33}} - \frac{\kappa_2^2}{\mu_{33}} & \varepsilon_{12} - \frac{\varepsilon_{13}\varepsilon_{32}}{\varepsilon_{33}} + \frac{\kappa_1\kappa_2}{\mu_{33}} & (-\frac{\mu_{31}}{\mu_{33}} + \frac{\varepsilon_{13}}{\varepsilon_{33}})\kappa_2 & -\frac{\mu_{32}\kappa_2}{\mu_{33}} - \frac{\varepsilon_{13}\kappa_1}{\varepsilon_{33}} \end{bmatrix} F(z)$$

wnere

$$\kappa = \frac{\omega}{c},\ \kappa_1 = \frac{k_1}{\kappa},\ \kappa_2 = \frac{k_2}{\kappa},\ c = 1, \text{ and } F(z) = \begin{bmatrix} E_1(z) \\ E_2(z) \\ H_1(z) \\ H_2(z) \end{bmatrix}$$

The solution in each layer is a combination of four mod

$\bar{v}e^{\lambda z}$, where v is an eigenvector and λ is an eigenvalue: $\bar{v}$

Ir $F(z) = c_1\bar{v}_1\varepsilon^{\lambda_1 z} + c_2\bar{v}_2\varepsilon^{\lambda_2 z} + c_3\bar{v}_3\varepsilon^{\lambda_3 z} + c_4\bar{v}_4\varepsilon^{\lambda_4 z}$

$$c^l = \begin{bmatrix} c_1^l & c_2^l & c_3^l & c_4^l \end{bmatrix}$$

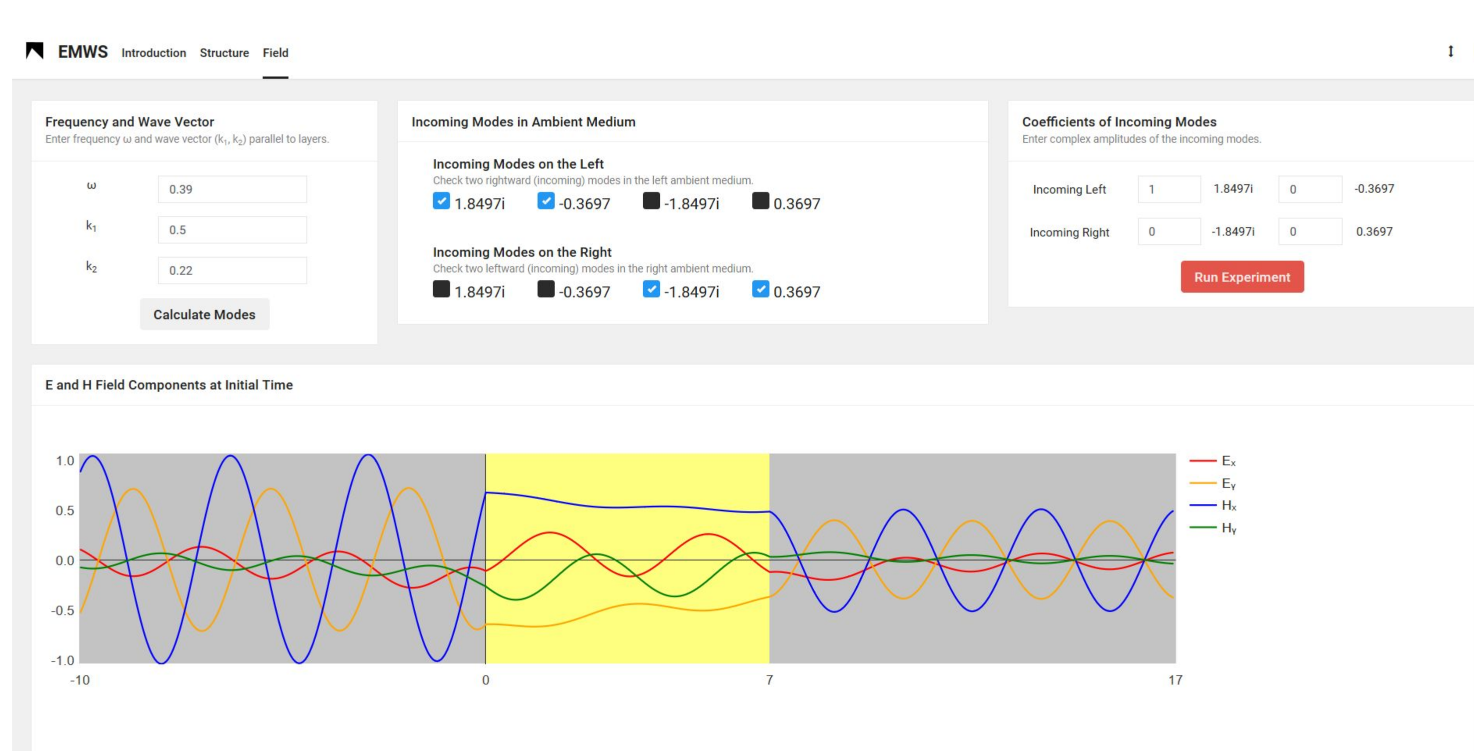
Continuity at the l^{th} interface is expressed as $n^+_l c^l = n^-_{l+1} c^{l+1}$.

This gives an 4(*n*-1) *x* 4*n* homogeneous linear system:

$$(n=6) \quad \begin{bmatrix} n_1^+ & -n_2^- & 0 & 0 & 0 & 0 \\ 0 & n_2^+ & -n_3^- & 0 & 0 & 0 \\ 0 & 0 & n_3^+ & -n_4^- & 0 & 0 \\ 0 & 0 & 0 & n_4^+ & -n_5^- & 0 \\ 0 & 0 & 0 & 0 & n_5^+ & -n_6^- \end{bmatrix} \begin{bmatrix} c_1^1 \\ c_2^1 \\ c_3^1 \\ c_4^1 \\ \vdots \\ c_1^6 \\ c_2^6 \\ c_3^6 \\ c_4^6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For the ering prc This results in a nonhomogeneous 4*n* *x* 4*n* linear system. Buy solving for all the coefficients we obtain the full field in the structure.

Graphical User Interface

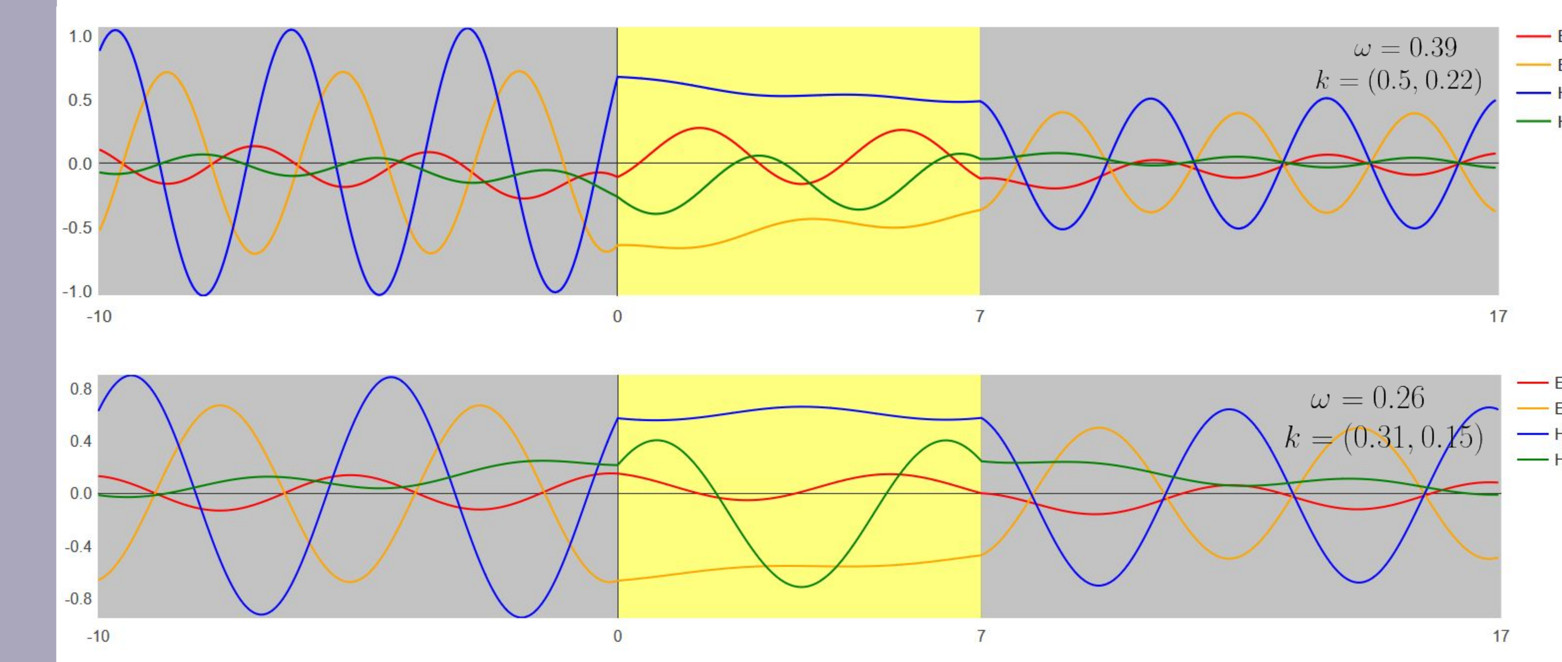


Screenshot of current graphical user interface (GUI)

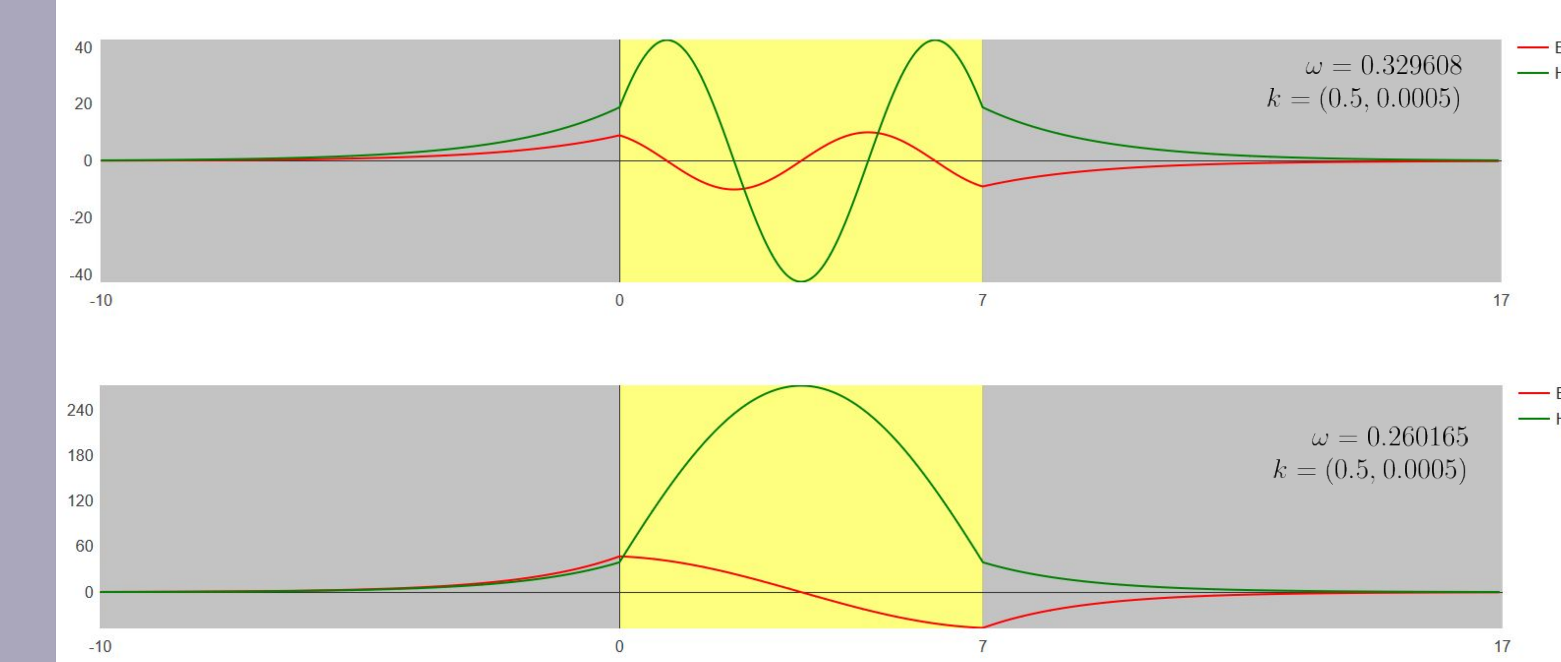
https://www.math.lsu.edu/~shipman/EMWS/html/dashboard.4.html

Simulations

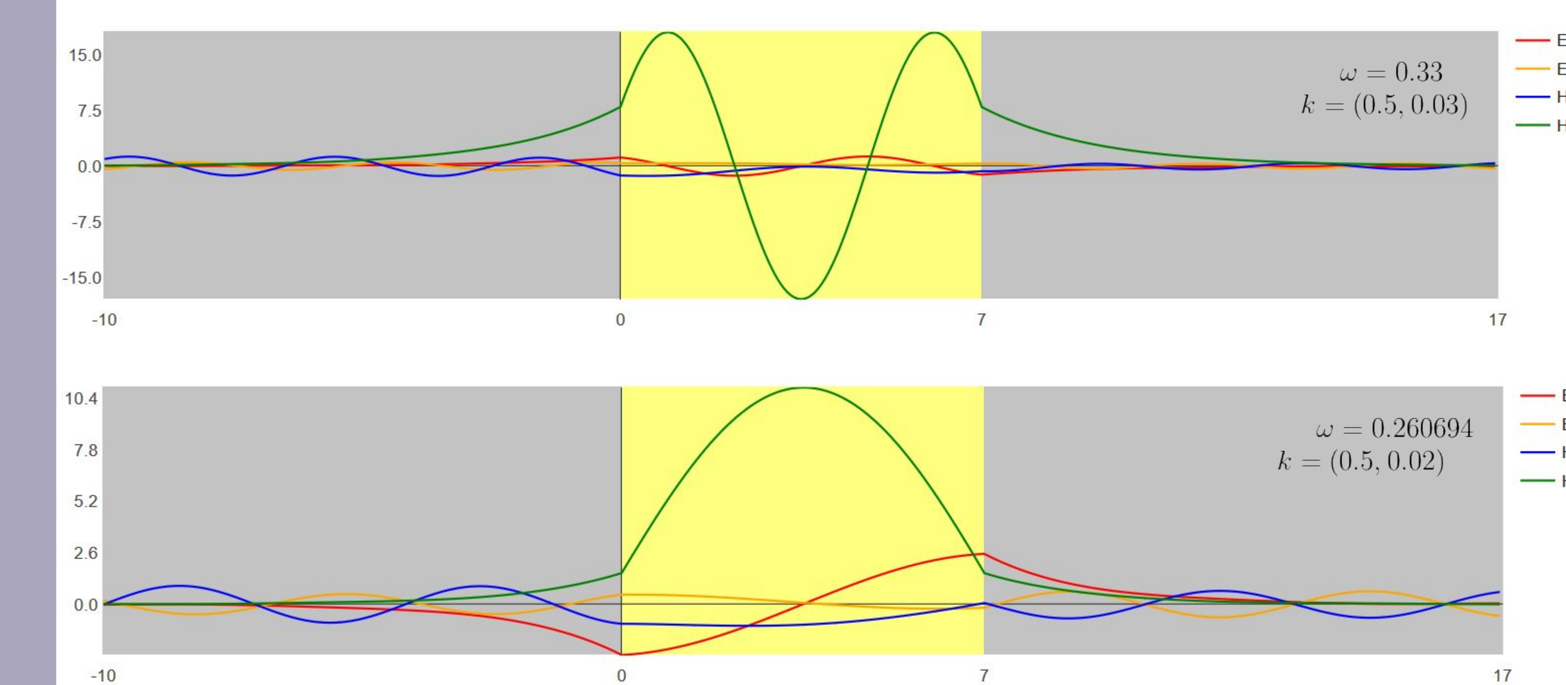
- ***Scattering***



- ***Guided Modes***



- ***Resonance***



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