

**2007 LSU Math Contest
Team Problems**

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and fifteen minutes to complete the entire team session.

These 10 questions (except for question 3) require exact numerical or algebraic answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. **The tiebreaker for the team competition is time.** If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.

1 Simplify the expression

$$\sqrt{\frac{8^{10} + 4^{10}}{8^4 + 4^{11}}}$$

2 Arrange the digits 1, 1, 2, 2, 3, 3, as a six-digit number in which the 1s are separated by one digit, the 2s are separated by two digits, and the 3s are separated by three digits. There are two answers. Find both.

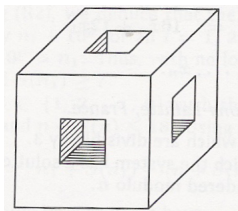
3 If $x/y = 3/4$, then which of the following expressions are incorrect:

A $\frac{x+y}{y} = \frac{7}{4}$ B $\frac{y}{y-x} = \frac{4}{1}$ C $\frac{x+2y}{x} = \frac{11}{3}$

D $\frac{x}{2y} = \frac{3}{8}$ E $\frac{x-y}{y} = \frac{1}{4}$

4 Consider a cube of edge 3 inches.

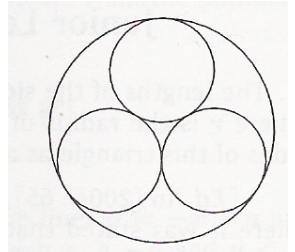
In the center of three different (and not opposite) faces we bore a square perforation of side 1 inch that goes across the cube as far as the opposite face. We thus obtain the following figure:



Determine the surface area of the resulting solid.

5 x men work x hours a day for x days to dig a tunnel of length x yards. If y men work y hours a day for y days what length (in yards) of the continuation of the tunnel would you expect them to dig?

6 Given three distinct unit circles (i.e. circles of radius 1), each of which is tangent to the other two, find the radius of the circle which is tangent to all three circles and contains them.



7 Find all real numbers x which satisfy

$$\sin x + \cos x = \sqrt{\frac{2 + \sqrt{3}}{2}},$$

with $0 < x < \frac{\pi}{2}$.

8 Let $a > 1$ and $b > 1$ be real numbers such that

$$\log_{10}(a + b) = \log_{10} a + \log_{10} b.$$

What can you say about the value of

$$\log_{10}(a - 1) + \log_{10}(b - 1) ?$$

9 Compute the integer k , $k > 2$, for which

$$\log_{10}[(k - 2)!] + \log_{10}[(k - 1)!] + 2 = 2 \log_{10}(k!).$$

10 On a certain island there are two varieties of people: the *truth tellers*, who always tell the truth and the *liars*, who always lie. You meet a group of three people there. Some of the three are liars and some are truth tellers, but you don't know who is who. You ask the first of them "What variety are you?" The first person answers, but you can't hear the answer. The second of the three then says: "The first one said he is a truth teller." Then the third one says: "The second one is lying." Determine what the second and the third are.