2008 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 18 are worth 1 point each and questions 19 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 18 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1
Simplify
$$\frac{2^{2008} + 2^{2006}}{2^{2007} + 2^{2005}}.$$

A 0   B 1   C 2   D 4   E none of these

2
Solve for x in $3x + 7 = 1 - 2x$.

A $-\frac{6}{5}$ B $\frac{1}{5}$ C $\frac{6}{5}$ D $-\frac{7}{6}$ E $\frac{7}{6}$

3
The function $f : \mathbf{R} \to \mathbf{R}$ given by the formula $f(x) = \sqrt{x^2}$ can also be written as

A $f(x) = x$ B $f(x) = |x|$ C $f(x) = (\sqrt{x})^2$ D $x \cdot \frac{x}{|x|}$
E none of the above

4
The sum of the degree measures of the interior angles in a quadrilateral is

A 180   B 360   C 540   D 720   E none of these

5
What is one-third of 199?

A 33   B 66   C 66.5   D $66\frac{2}{3}$ E $66\frac{1}{3}$

6
Which of these numbers is the largest?

A $\frac{1}{3}$ B $\frac{3}{10}$ C $\frac{333}{1000}$ D $\frac{7}{20}$ E 0.33

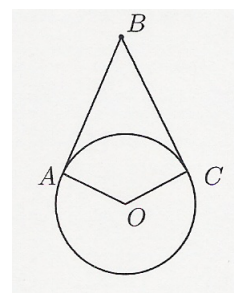**7**
How many axes of symmetry does a non-square rhombus have?
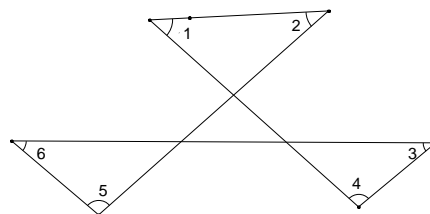
A 0   B 1   C 2   D 3   E 4

8
In the diagram on the right, O is the center of the circle, $\overline{AB}$ and $\overline{BC}$ are tangent lines to the circle with points of tangency A and C respectively. If $\angle O$ measures $90°$ then the degree measure of $\angle B$ is

A $45°$ B $60°$ C $90°$ D $120°$ E None of the choices

9
The sum of the marked angles 1, 2, 3, 4, 5, 6 is

A $180°$ B $270°$ C $360°$ D $450°$ E can't be sure

10
From 1900 to 1950 the population of East Baton Rouge Parish increased 4920% and from 1950 to 2000 the population increased 150%. What was the percentage increase during the 20th century?

A $4920 + 150$ B $(4920)(150)$ C $\left(1 + \frac{4920}{100}\right)\left(1 + \frac{150}{100}\right) - 1$

D $100\left(1 + \frac{4920}{100}\right)\left(1 + \frac{150}{100}\right) - 100$

E $(100 + 4920)(100 + 150) - 100$

11
Let $f(n) = n/((n-1)/\cdots/(3/(2/1))\cdots)$ for $n = 2, 3, \ldots$. Which of the following is true?

A $f(7) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ B $f(7) = \frac{7 \cdot 5 \cdot 3}{6 \cdot 4 \cdot 2}$ C $f(7) = \frac{6 \cdot 4 \cdot 2}{7 \cdot 5 \cdot 3}$

D $f(7) = 1$ E none of these

12

If the operation $\diamondsuit$ is defined as $a \diamondsuit b = a^{b-1}$ for $a, b > 0$, then $3 \diamondsuit (2 \diamondsuit 3)$ is

A 27   B 9   C 3   D 1   E none of these

13

If the length of a rectangle is increased by 30% and the width is decreased by 20%, then the area is

     A increased by 4%

     B decreased by 6%

     C increased by 6%

     D increased by 10%

     E increases by 56%

14

Suppose the equation $x^2 + bx + 4 = 0$ has exactly one solution. The best statement about b is:

     A b must be equal to 4

     B b must be equal to -4

     C b must be equal to 2

   D b must be equal to -2

     E none of the above

15

Of the following statements, the one that is incorrect is:

     A Doubling the base of a rectangle doubles the area.

     B Doubling the altitude of a triangle doubles the area.

     C Doubling the radius of a circle doubles the area.

     D Doubling the denominator of a positive fraction and dividing its numerator by 2 changes the fraction.

     E Doubling a given number may make it less than it originally was.

16

Suppose the triangle $\triangle ABC$ shown in the diagram is an equilateral triangle. If the line BC is described by an equation of the form $y = mx + b$ for some m and b, then find m.
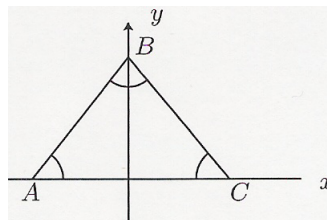
A 60   B 90   C $-1/2$ D $-\sqrt{3}$ E -60

17

During the final game of a basketball tournament, only 7 players from the tournament winning team played. The scoring average of all 7 players was 13. The scoring average of everyone but the point guard was 12. How many points did the point guard score?

A 14   B 16   C 19   D 21   E 24

18

A small circle just fits inside a semicircle. What is the ratio of the area of the small circle to the area of the semicircle containing it?
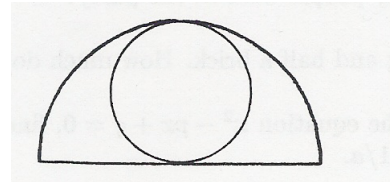
A 1:1   B 1:2   C 1:3   D 2:3   E 3:4

Questions 19 - 28 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

19 An airport has a moving sidewalk. Standing on the moving sidewalk, Joe takes 3 minutes to reach the end. Walking next to the moving sidewalk, Joe takes 4 minutes to reach the end. How long does it take Joe to reach the end if he walks on the moving sidewalk?

20 John earned some money. When he was paid, his employer had already deducted 20% federal and 2% state income taxes. He was paid $792. How much, to the nearest dollar, did he earn?

21 A six place number is formed by repeating a three place number: For example, 256,256, or 678,678, etc. Call these numbers *copycats*. Find greatest common divisor of all the copycats.

22 The squares $ABCD$ and $EFGH$ have equal areas. The area of the shaded square is 1/9 of the area of the square $ABCD$. Find the area of the square $ARGS$ if the area of the shaded square is $49in^2$.
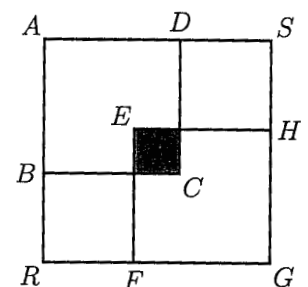
23 For how many different positive integers n does $\sqrt{n}$ differ from $\sqrt{16}$ by less than 1?

24 What is the last digit of 2^{2008}?

25 Let

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{2008^2}\right) = \frac{x}{2 \cdot 2008}.$$

Compute the integer x.

26 A line is called a supporting line of a figure if the line meets the figure but the whole figure is on one side of the line. For example, the line that contains the side of a square is a supporting line of the square. Consider the figure below drawn on the coordinate plane - it is half of the unit circle.
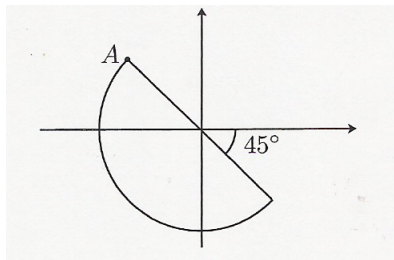
If a supporting line passes through A, then how large of slope can the line have and how small of slope can the line have?

27 Suppose A and B are points on the circle with center C. The angle $\angle ACB$ is $30°$. If a point D is randomly chosen on the circle, then what is the probability that the triangle $\triangle ABD$ is obtuse?

28 A room is 30 feet square and 12 feet high. A spider is located in one of the corners on the floor. An unsuspecting fly rests at the diagonally opposite corner on the ceiling. If the fly does not move, what is the shortest distance the spider must crawl, always touching a surface of the room, to catch the fly?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **28** above.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.