

2012 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 21 Multiple Choice

Please:

- Use the answer sheet for your answers.
- Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.
- Completely erase any answer you wish to change.
- Do not make stray marks on the answer sheet.

1
 $\sqrt{3^{100}} =$
A 300 B $(\sqrt{3})^{10}$ C 3^{50} D 3^{200} E None of these

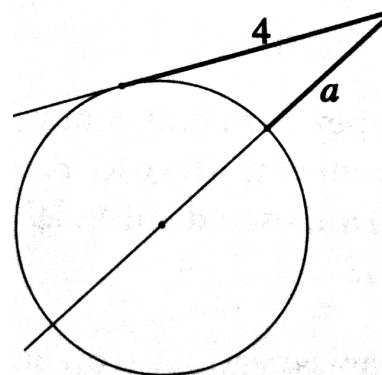
2
If $\sqrt{2}(x - 1) = x$, then $x =$
A $2 + \sqrt{2}$ B $\sqrt{2}$ C $2 - \sqrt{2}$ D 0 E None of these

3
The solution to the inequality $4 - 5x < 3x + 1$ and the smallest integer satisfying this inequality are
A $x > 0.375$; 1
B $x < 0.375$; no such integer exists
C $x = 0.375$; no such integer exists
D $x > 2$; 3
E $x < 2$; no such integer exists

4
Ancient Babylonians used $3\frac{1}{8}$ as the number π . Round up $\pi = 3.1415\dots$ with 0.001 accuracy, and check by how many percent does it differ from $3\frac{1}{8}$. Round up the result to 0.01%.
A 0.48% B 0.5% C 0.53% D 0.54% E 0.55%

5
Sum of all two-digit natural numbers is
A 4850 B 4905 C 5450 D 9810 E none of these

6
The circle in the picture



has radius 2. The length of the marked segment a is

A $2\sqrt{5} + 2$ B 4 C $2\sqrt{5} - 2$ D $2\sqrt{5}$ E none of these

7
Two different numbers y and z are roots of the quadratic equation $ax^2 + c = 0$. Which one of the following is not true in general
A $y + z = 0$ B $y^2 + z^2 = 0$ C $y^3 + z^3 = 0$
D $y/z = -1$ E $y \cdot z < 0$

8
All the vertices of a convex quadrilateral $ABCD$ lie on two perpendicular lines. It follows that
A A circle can be inscribed in the quadrilateral $ABCD$
B $ABCD$ can be inscribed in a circle
C $ABCD$ is a square
D $ABCD$ is a rhombus
E None of the above is true

9
The length of the earth's equator is approximately 40000 km. What is the approximate length of the 60° parallel?
A 20000 B 23500 C 26200 D 30000 E 34000

10
If the greatest common divisor of two natural numbers a and b is 3 and $\frac{a}{b} = 0.4$, then ab is
A 10 B 18 C 30 D 36 E 90

11
How many different real roots does the polynomial $x^4 - x^3 - x^2 + x$ have?
A none B one C two D three E four

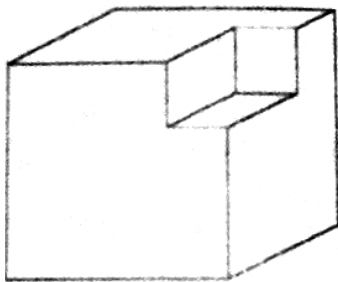
12

Every other day Alice only tells the truth, on the remaining days she always lies. Today she said exactly four of the five statements below. Which is the one she didn't say?

- A The number of my friends is a prime number.
- B Half of my friends are boys.
- C 288 is divisible by 12.
- D I always tell the truth.
- E Three of my friends are older than me.

13

A box shaped part was removed from a corner of a wooden cube of volume 512 in^3 , as shown on the picture below.



What is the surface area of the resulting solid?

- A 320 in^2
- B 336 in^2
- C 384 in^2
- D 468 in^2
- E not enough information to determine

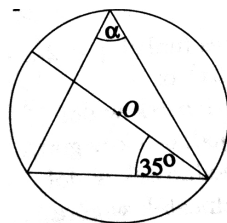
14

$$\sqrt{20} + \sqrt{45} + \sqrt{80} - 2\sqrt{125} + \frac{1}{\sqrt{5}-2} =$$

- A $\sqrt{5} + 2$
- B 3
- C $2\sqrt{5} - 1$
- D 2
- E 5

15

In the diagram on the right, O = center of circle, the angle α is



- A 35°
- B 50°
- C 55°
- D 60°
- E None of these

16

The numbers w, x, y, z form an arithmetic progression with difference d . If $w + x = s$, then $y + z =$

- A $s + 2d$
- B $s + 3d$
- C $s + 4d$
- D $s + 5d$
- E None of these

17

For which non-zero real numbers x is the number $\frac{|x - |x||}{x}$ a positive integer?

- A only for negative x

- B only for positive x
- C only for even integers x
- D for any non-zero real number x
- E no such real x exists

18 [This is also a tie breaker question - see page 3.]

Write ten consecutive prime numbers next to each other from left to right starting with 2. In the multi-digit number so obtained, delete half of the digits so that the resulting number is the greatest possible of all numbers obtained that way. What is the fifth digit from the left of the resulting number?

- A 7
- B 5
- C 3
- D 2
- E 1

19

Let a and b be real numbers with $a < b$.

Which of the following sets

- A $(-\infty, a)$
- B $(b, +\infty)$
- C (a, b)
- D all the real numbers
- E empty set

is a set of all the solutions of the inequality

$$\left| x - \frac{a+b}{2} \right| < \frac{b-a}{2} ?$$

20

How many integer solutions does the equation

$$x(x+1) + (x+1)(x+2) + \dots + (x+9)(x+10) = 1000x + 2011$$

have ?

- A 0
- B 1
- C 2
- D 10
- E infinitely many

21

The shortest side of a triangle has length 1 and no side is longer than 3. What is the largest area of a triangle satisfying this condition?

- A $\frac{3}{2}$
- B $\frac{\sqrt{10}}{2}$
- C $\frac{\sqrt{33}}{4}$
- D $\frac{\sqrt{35}}{4}$
- E $\frac{\sqrt{37}}{4}$

Questions 22 - 28 Exact Answers

These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40} .

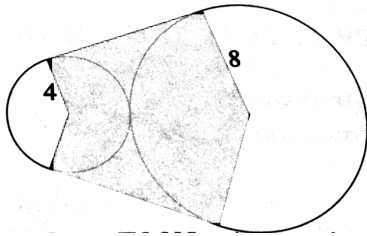
22 Water when freezing (and turning into ice) enlarges its volume by a factor of $\frac{1}{10}$. By what factor will the volume of ice decrease when it is melted?

23 When Achilles started chasing a turtle they were 990 yards apart. Achilles covers 10 yards in each second. The turtle covers 1 yard in 10 seconds. How long will it take Achilles to catch the turtle?

24 For which natural number n is the distance between the numbers $2^0 + 2^1 + 2^2 + \dots + 2^n$ and 2012 the smallest?

25 A digital clock has a special button B. Pushing that button causes the clock to switch to the next exact hour, i.e. it either moves back 0 to 29 minutes or moves forward from 1 to 30 minutes. If at 10:15 the clock shows 8:20, what is the shortest time needed to set the clock to the right time by only pushing B, possibly a few times. The clock runs accurately - doesn't slow or go fast.

26 Find the area of the shaded region.



27 The numbers, $2a - 2$, $2a + 2$, and $a + 1$ are lengths of three sides of a triangle. What can you say about the number a ?

28 A polyhedron has n faces, one of which is a regular pentagon. What is the minimal possible value of n ?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question 18.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.