

Questions 1 - 13 are worth 1 point each and questions 14 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

The people supervising this test are not permitted to explain to you the meaning of any question.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 13 Multiple Choice

Please:

- Use the answer sheet for your answers.
- Answer only one choice A, B, C, D, or E for each question by writing your answer on the answer sheet.
- Completely erase any answer you wish to change.
- Do not make stray marks on the answer sheet.

1. Which of the following would be the best response to

$$\sqrt{-1} 2^3 \sum \pi?$$

- A and then my wife left me.
- B but it was the wrong formula
- C so we turned to the right
- D but I thought the contest was today
- E and it was delicious

2. On a certain trading day Stock XXX declined \$.27 or 4% of its value. What was the value of the stock at the end of the trading day?

- A \$6.75
- B \$6.48
- C \$6.90
- D \$7.10
- E \$7.02

3. Mike has 40 coins consisting of dimes and quarters. If the dimes were quarters and the quarters dimes he would have 90 cents more. How many quarters does he have?

- A 17
- B 19
- C 23
- D 25
- E 27

4. The equation $y - k = a(x - h)^2$ has graph a parabola with vertex (h, k) . Suppose $a > 0$. A horizontal line 1 unit above the vertex intersects the graph at two points. Find the distance between these two points.

- A $2h$
- B $2k$
- C $2\sqrt{a}$
- D $\frac{2\sqrt{a}}{a}$
- E $\frac{2}{a}$

5. Given that $\log_{10} 2 = .30103$, rounded to five decimal places, how many digits are there in the decimal expansion of 2^{2019} .

- A 67
- B 607
- C 68
- D 608
- E 69

6. The graph of the equation

$$x^2 - 8x + y^2 - 6y = 75$$

is a circle with what center?

- A $(0, 0)$
- B $(4, 3)$
- C $(3, 4)$
- D $(8, 6)$
- E $(6, 8)$

7. How many four-digit numbers between 7000 and 8000 are there for which the thousands digits equal the sum of the other three digits.

- A 10
- B 24
- C 36
- D 72
- E none of these

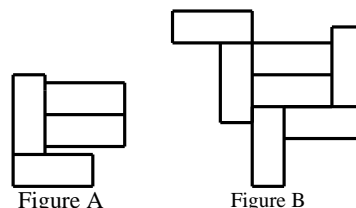
8. If S is the set of all points in the plane whose distance to the origin is twice the distance to the point $(4, 4)$ then S is a

- A circle
- B ellipse
- C parabola
- D hyperbola
- E the empty set

9. What is the remainder when $(2019)^4$ is divided by 7?

- A 1
- B 2
- C 3
- D 4
- E 6

10. Figures A and B below are made up of congruent rectangles each of which has perimeter 13. If the perimeter of Figure A is 31 then what is the perimeter Figure B?

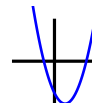


- A 38
- B 40
- C $42\frac{1}{2}$
- D 44
- E $44\frac{1}{2}$

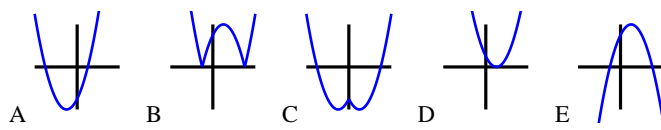
11. Suppose x and y are positive integers such that $(x + y)^2 + 3y^2 = 63$. Find x .

- A 1
- B 2
- C 3
- D 4
- E 6

12. Suppose $y = f(x)$ has the following graph:



Which of the following is the graph of $y = f(|x|)$?



13. Suppose a and b are positive numbers such that

$$a^2 - b^2 = 2ab = 4(a + b).$$

Find a .

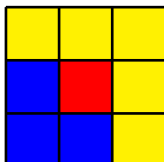
- A 4
- B $4 + \sqrt{8}$
- C $\sqrt{8} - 2$
- D $6 + \sqrt{8}$
- E 8

Questions 14 - 24 Exact Answers

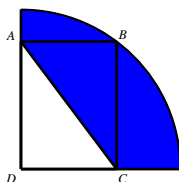
These next eleven questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40} .

14. In a certain "3-2-1" course an athlete must cycle 3 miles, then run 2 miles, and finally swim 1 mile. He can cycle 4 times faster than he can run and he can run 3 times faster than he can swim. If it takes 34 minutes less time to cycle 6 miles than to complete the "3-2-1", how long does it take him to complete the "3-2-1" course?

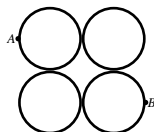
15. Suppose (x,y) is on the circle of radius 4 centered at the origin. What is the largest possible value of $x^2 + 4y^2 + 6x + 1$?
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16. At a certain food fair a game of darts is played with a board that is a 3×3 square grid with three color regions: red, blue, and yellow. If a dart lands on red you win 5 apples, if it lands on yellow you win 1 apple, and if it lands on blue you win nothing. You must donate an apple for each dart you throw. Suppose your dart throwing ability is such that your dart always hits the board and it is equally likely that it lands on any square. You come with a basket of nine apples and purchase nine darts with them. After throwing all your darts how many apples can you expect to walk away with?



17. Rectangle $ABCD$ is inscribed in a quarter circle of radius 6 with AD and DC lying on a radius. See the diagram below. Suppose $AD + DC = 8$. Find the perimeter of the shaded region.



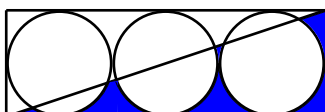
18. Four circles are arranged as in the diagram below. Neighboring circles are tangent.



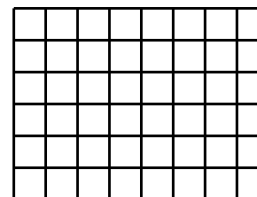
How many paths are there from A to B along the arcs of the circles if you are not allowed to meet any tangent point more than once?

19. You play the following game with a friend. You share a pile of chips, and you take turns removing between one and seven chips from the pile. (In particular, at least one chip must be removed on each turn.) The game ends when the last chip is removed from the pile; the one who removes it is the loser. It is your turn, and there are 2019 chips in the pile. How many chips should you remove to guarantee that you win, assuming you then make the best moves until the game is over?

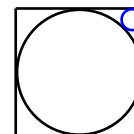
20. Three non overlapping circles of radius 2 sit in 4×12 rectangle. A diagonal is drawn. See the diagram below. Find the area of the region shaded blue. That is, the area of the region in the lower triangle but outside of the circles.



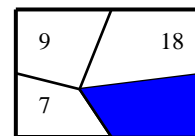
21. While taking a hike in the "geometric mountain range" you come across **cylinder mountain**, so named because it is a perfect cylinder. Astonished, you begin to investigate. You hike around the base of cylinder mountain and find that it is exactly a 5 mile walk. At one point at the base you notice a trail to the top that wraps one time around the outside of the mountain. The trail sign says the length of the trail is 10 miles and you walk at a constant upward pitch. When you reach the top you stand exactly above the point where you started at the base. What is the height of cylinder mountain?
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22. How many **squares** can be formed from the grid lines in the following figure given that all grid lines are equally spaced.



23. A circle is inscribed in a square of side length 2. In one corner a circle is inscribed in the region outside of the circle but within the square. Find its area. See the diagram below.



24. In a certain rectangle a point is chosen. A line segment is drawn from the point to the midpoint of each side of the rectangle and thus partitions the rectangle into 4 regions. The areas of 3 of the regions is given in the diagram below. What is the area of the fourth region- the one shaded blue?



Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 24**. Write your explanation on the **reverse side** of your answer sheet.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.