Seminar Notes **June 3, 2011**

Yesterday, you wrote out solutions to the problem below, and four people put four different solutions (summarized after the problem statement) on the board. Today, we will revisit these solutions, seeking analogies. If you look carefully enough, you will be able to see part-by-part correspondences between them. Each meaningful piece of each solution is represented in a different way and by different images or symbols in all the other solutions. (For example, the 15 mph that appears in Solution 4 is the rate at which the vertical distance between the graphs that appear in Solution 3 is changing.)

Problem. A pink Cadillac leaves Oklahoma City at 6AM headed west on I-40 with the cruise control set at 70 mph. A federal agent in a Toyota follows, leaving at 7AM and traveling 85 mph. When does the Toyota catch up?

Solution 1. The following table shows the time, the number of hours each car has been traveling and the miles west of Oklahoma City:

time	hours traveled (Cadillac)	hours traveled (Toyota)	distance traveled (Cadillac)	distance traveled (Toyota)
6 : 00	0	0	0	0
7 : 00	1	0	70	0
8 : 00	2	1	140	85
9 : 00	3	2	210	170
10 : 00	4	3	280	255
11 : 00	5	4	350	340
11 : 10	$5\frac{1}{6}$	$4\frac{1}{6}$	$361\frac{2}{3}$	$354\frac{1}{6}$
11 : 20	$5\frac{1}{3}$	$4\frac{1}{3}$	$373\frac{1}{3}$	$368\frac{1}{3}$
11 : 30	$5\frac{1}{2}$	$4\frac{1}{2}$	385	$382\frac{1}{2}$
11 : 40	$5\frac{2}{3}$	$4\frac{2}{3}$	$396\frac{2}{3}$	$396\frac{2}{3}$

We see that the distance traveled by the Cadillac is equal to the distance travelled by the Toyota at 11 : 40, so this is the time at which the Toyota catches up.

Solution 2. Let t be the number of hours elapsed since 6AM. Let d_C be the distance travelled by the Cadillac and let d_T be the distance travelled by the Toyota. Then:

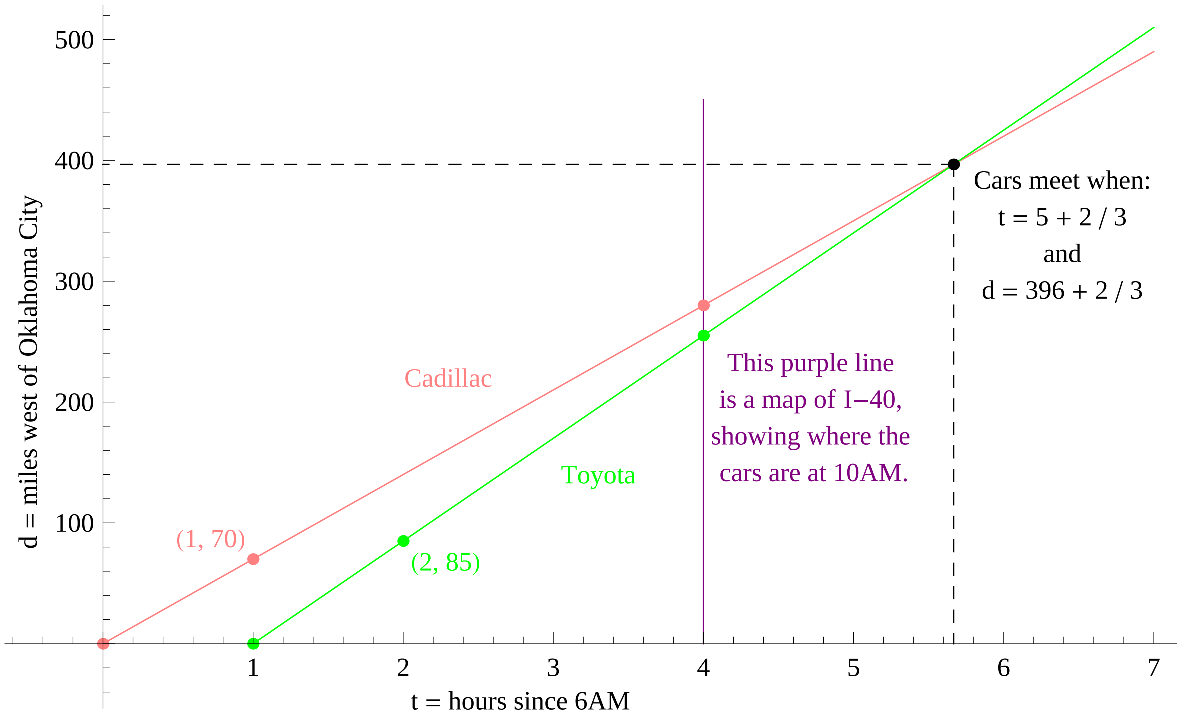
$$d_C = 70t;$$
$$d_T = 85(t-1).$$

We need to find the time when $d_C = d_T$. If we determine the number of hours elapsed (i.e., the value of t) when this occurs, we can easily find the time. Now,

$$\begin{aligned} d_C = d_T \;&\Leftrightarrow\; 70t = 85(t-1) \\ &\Leftrightarrow\; t = 5\frac{2}{3} \end{aligned}$$

Thus, the cars meet $5\frac{2}{3}$ hours after 6AM, which is 11 : 40.

Solution 3.



Solution 4. At 7AM the cars are 70 miles apart and after 7AM until they meet, the distance between them is closing at 15 mph. It takes 70/15 hours for the distance between the cars to go to zero. Since $70/15 = 4\frac{2}{3}$, he cars meet $4\frac{2}{3}$ hours after 7AM, which is $11:40$.

In discussions in the seminar, we made the following observations.

A. Time is represented in different (but analogous) ways in the various solutions:
   - S-1. Here, the first column shows clock time. Each row describes conditions that are true at the specific time noted in the first column.
   - S-2. Time is represented by the variable t, which can be assigned different values.
   - S-3. All possible times are represented on the horizontal axis; the relative positions of the two cars at a specific time are represented by the points where the pink and green graphs cross the vertical line through that point on the horizontal axis that represents that time.
   - S-4. This solution is so compressed that it is hard to see the time explicitly. We divide rate by distance to get the amount of time it takes to make a change in relative position.

B. Time is the *independent variable*. There are two varying positions (one for each car), and these are *dependent variables*. The problem is solved by finding the time which makes the dependent variables equal (or in solution 4, makes the difference between them vanish).

C. Position is also is represented in different (but analogous) ways in the various solutions. There are *two* positions to keep track of, except in S-4, where the distances are combined in one idea, the distance between the cars.
   - S-1. The positions of the cars at the times listed in column 1 are shown in two columns.
   - S-2. There are two variables representing the positions, d_C and d_T.
   - S-3. All positions of the Toyota at all times are shown by the green graph; the specific position at a specific time is seen on a vertical line.