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Let f be a convex function of n variables, bounded below, with infimum attained. A bracket is an interval $[L,U]$ containing the attained infimum $\min f$. The Newton Bracketing (NB) method [1] is an iterative method that at each iteration does a single Newton iteration and then reduces the bracket by either raising L or lowering U . An initial lower bound L must be provided, and the initial upper bound is $U(x)$ where x is the initial iterate.

Unlike gradient methods (and other iterative methods that directly drive x to optimum), the NB method has a natural stopping criterion: the bracket size. Numerical experience reported here shows the average bracket reduction per iteration is about $1/2$, so convergence is fast.

The method is valid for $n = 1$. It is valid for $n > 1$ (using the directional Newton iteration [2]) provided the level sets of f are not "too narrow". A precise statement for quadratic f , i.e., $f = \frac{1}{2}x^T Qx + \text{linear terms}$, with Q positive definite, is: the NB method is valid if the condition number of Q is $< 1/(7 - \sqrt{48})$ (~ 14).

The method was applied in [1] for solving location (Fermat-Weber) problems, and is applied here to location problems with affine constraints, and to affinely constrained convex programs in general. Applications to trust region subproblems in SDP are developed by H. Wolkowicz.

[1] Y. Levin and A. Ben-Israel, "The Newton Bracketing Method for Convex Minimization," *Computational Optimization and Applications*, **21**(2002), pp. 213-229.

[2] Y. Levin and A. Ben-Israel, "Directional Newton Methods in n Variables," *Math. of Computation*, **71**(2002), pp. 237-250.