MATH 2065-4, Fall 2010 Name: ______________________

Test 1, Thursday, Sept 23, 2010

. For partial credit, show all your work!

1[6P]) Which of the following functions $y(t)$ is a solution (**S**)/not a solution (**N**) to the differential equation $y' + 2ty = t$?

$y(t)$	Y	N
$1/2$		
$e^t + 1/2$		
$2e^{-t^2} + 1/2$		

2[9P]) Determine and mark with **Y** for yes, and **N** for no, if each of the following differential equation is separable (**S**), linear (**L**), and/or homogeneous (**H**). Note, that in each case, more than one might be correct.

Equation	S	L	H
$t^2y' = y^2 + ty$			
$y' - t^2y = t^3y$			
$(y^2 + 2tyy' = ty$			

3) Solve the following differential equation:

a[10P]) $y' + 2ty = 0$. **Solution** $y(t) =$ ____________

b[10P]) $y' + y = ty^2$. **Solution** $y(t) =$ ____________

5) Solve the following initial value problems.

a[15P]) $y' - \frac{1}{2}ty = t^3 e^{t^2}$, $y(0) = 3$. **Solution** $y(t) =$ ____________

b[15P]) $t^2 y' = -y^2$, $y(1) = 1$. **Solution** $y(t) =$ ____________

3) A tank contains 100 gal of brine made by dissolving 40 lb of salt in water. A brine solution containing 10 grams salt per liter flows into the container at a rate of 4 liters per minute. The well stirred mixture runs out at the same rate. Denote by $y(t)$ the amount of salt in the tank at time t.

a[8P])) Write an initial value problem that $y(t)$ must satisfy.

Solution: ____________________

b[10P]) Solve the initial value problem. **Solution:** $y(t) =$ ____________

c[5P]) How much salt is in the tank after 10 min? **Solution:** ____________

4[12P]) Apply Picard's to compute the approximations $y_0(t)$, $y_1(t)$, and $y_2(t)$ to the solution of the initial value problem $y' = (y+1)^2, \quad y(0) = 0$.

MATH 2065-4, Fall 2010 Name: ______________________

Test 1, Thursday, Sept 23, 2010

. For partial credit, show all your work!

1[6P]) Which of the following functions $y(t)$ is a solution (**S**)/not a solution (**N**) to the differential equation $y' = y - t$?

$y(t)$	Y	N
$t+1$		
$e^t + 1 + t$		
$2e^t - t - 1$		

2[9P]) Determine and mark with **Y** for yes, and **N** for no, if each of the following differential equation is separable (**S**), linear (**L**), and/or homogeneous (**H**). Note, that in each case, more than one might be correct.

Equation	S	L	H
$y' = \dfrac{y-t}{y+t}$			
$y' - ty = t^3 y$			
$(y^2 + 2t^2) + 2tyy' = 0$			

3) Solve the following differential equation:

a[10P]) $y' = -2yt$. **Solution** $y(t) =$ ______________

b[10P]) $y' - y = ty^2$. **Solution** $y(t) =$ ____________

5) Solve the following initial value problems.

a[15P]) $y' - \frac{3}{t}y = t^3 e^t$, $y(1) = 3$. **Solution** $y(t) =$ ____________

b[15P]) $t^2 y' = -y^2 + yt$, $y(1) = 1$. **Solution** $y(t) =$ ____________

3) A tank contains 100 gal of brine made by dissolving 40 lb of salt in water. Pure water runs into the tank at the rate of 4 gal/min, and the mixture. which is kept uniform by stirring, runs out at the same rate. Denote by $y(t)$ the amount of salt in the tank at time t.

a[8P])) Write an initial value problem that $y(t)$ must satisfy.

Solution: ____________________

b[10P]) Solve the initial value problem. **Solution:** $y(t) =$ ____________

c[5P]) How much salt is in the tank after 10 min? **Solution:** ____________

4[12P]) Apply Picard's to compute the approximations $y_0(t)$, $y_1(t)$, and $y_2(t)$ to the solution of the initial value problem $y' = y^2 + 1, \quad y(0) = 0$.

MATH 2065-4, Fall 2010 Name: SOLUTION

Test 1, Thursday, Sept 23, 2010

. For partial credit, show all your work!

1[6P]) Which of the following functions $y(t)$ is a solution (**S**)/not a solution (**N**) to the differential equation $y' = y - t$?

$y(t)$	Y	N
$t+1$	✓	
$e^t + 1 + t$	✓	
$2e^t - t - 1$		✓

$y = t+1,\ y' = 1,\ y - t = t+1-t = 1 = y'$

$y = e^t + t + 1,\ y' = e^t + 1,\ y - t = e^t + 1$

$y = 2e^t - t - 1,\ y' = 2e^t - 1,\ y - t = 2e^t - 2t - 1 \neq y'$

2[9P]) Determine and mark with **Y** for yes, and **N** for no, if each of the following differential equation is separable (**S**), linear (**L**), and/or homogeneous (**H**). Note, that in each case, more than one might be correct.

Equation	S	L	H
$y' = \dfrac{y-t}{y+t}$			✓
$y' - ty = t^3y$	✓	✓	
$(y^2 + 2t^2) + 2tyy' = 0$			✓

$y' = \frac{y-t}{y+t} = \frac{(y/t)-1}{(y/t)+1}\ \left(= \frac{v-1}{v+1}\right)$ H.

$y' = t^3y + ty = (t^3+t)y$ separable

$y' - (t+t^3)y = 0$, linear

standard form

$y' = \frac{y^2+2t^2}{2ty} = \frac{\left(\frac{y}{t}\right)^2 + 2}{2(y/t)}$ homogeneous

3) Solve the following differential equation:

a[10P]) $y' = -2yt$. **Solution** $y(t) =$ Ce^{-t^2}

you can use that this is separable and also linear $y' + 2ty = 0$.

separable

$\frac{dy}{y} = -2t\,dt,\ \ln|y| = -t^2 - c$

$y = Ce^{-t^2}$

b[10P]) $y' - y = ty^2$. Solution $y(t) = \dfrac{1}{-t+1+Ce^{-t}}$

Bernoulli equation with $n = 2$, $y^{-2}y' - \frac{1}{y} = t$.

$z = 1/y$, $z' = -\frac{1}{y^2}y'$

$z' + z = -t$, $p = 1$, $P(t) = t$, $\mu(t) = e^t$

$-\int te^t dt = -te^t + e^t + C$, $z = \frac{1}{\mu}(-te^t + e^t + C) = -t + 1 + Ce^{-t}$

$y = \frac{1}{z} = \dfrac{1}{-t+1+Ce^{-t}}$

5) Solve the following initial value problems.

a[15P]) $y' - \frac{3}{t}y = t^3e^t$, $y(1) = 3$. Solution $y(t) = t^3(e^t + 3 - e)$

Linear: $\mu = e^{-3\ln t} = t^{-3}$

$\int e^t dt = e^t + C$

$y(t) = t^3(e^t + C)$

$y(1) = e + C = 3$, $C = 3 - e$

b[15P]) $t^2y' = -y^2 + yt$, $y(1) = 1$. Solution $y(t) = \dfrac{t}{\ln(t)+1}$

This is a homogeneous equation

$y' = -\left(\frac{y}{t}\right)^2 + \frac{y}{t} = -v^2 + v$ if $v = \frac{y}{t}$

$tv' + v = -v^2 + v$ or $tv' = -v^2$

$-\frac{dv}{v^2} = \frac{dt}{t}$, $\frac{1}{v} = \ln(t) + c \; (t > 0)$

$v = \dfrac{1}{\ln(t)+C}$, $y = tv = \dfrac{t}{\ln(t)+C}$

$y(1) = 1 = \frac{1}{c}$ so $c = 1$

3) A tank contains 100 gal of brine made by dissolving 40 lb of salt in water. Pure water runs into the tank at the rate of 4 gal/min, and the mixture. which is kept uniform by stirring, runs out at the same rate. Denote by $y(t)$ the amount of salt in the tank at time t.

a[8P])) Write an initial value problem that $y(t)$ must satisfy.

Solution: $y' = -\frac{1}{25}y$ and $y(0) = 40$

$y' =$ input − output. There is no salt in the input so, output $\frac{4}{100}y(t)$

$y' = -\frac{1}{25}y$. It is given, that at time $t=0$ the amount of salt is 40lb.

b[10P]) Solve the initial value problem. **Solution:** $y(t) = 40e^{-\frac{t}{25}}$

$\frac{dy}{y} = -\frac{dt}{25}$, $\ln y = -\frac{t}{25} + C \quad (y>0)$

$y = Ce^{-t/25}$. Taking $t=0$, $y(0) = 40 = C$

c[5P]) How much salt is in the tank after 10 min? **Solution:** $40e^{-\frac{2}{5}} \approx 26.8$

$y(10) = 40e^{-\frac{10}{25}} = 40e^{-2/5} \approx 26.8$ lb

4[12P]) Apply Picard's to compute the approximations $y_0(t)$, $y_1(t)$, and $y_2(t)$ to the solution of the initial value problem $y' = y^2 + 1, \quad y(0) = 0$.

$y_0 = 0$

$y_1 = 0 + \int_0^t 0^2 + 1\, du = \int_0^t du = t$

$y_2 = 0 + \int_0^t u^2 + 1\, du = \frac{1}{3}t^3 + t.$

MATH 2065-4, Fall 2010 Name: ____________________

Test 2, Thursday, Sept 26, 2010. For partial credit, show all your work!

1[24P]) Compute the Laplace transform of each of the following functions:

a) $\mathcal{L}\left(t^3e^{-2t}\right)(s) =$

b) $\mathcal{L}\left(t\sin(4t) + e^{-3t}\cos(t)\right)(s) =$

c) $\mathcal{L}\left((2te^t + 1)(t^3 + 2)\right)(s) =$

2[24P]) Find the partial fraction decomposition of each of the following rational functions. You may use the convolution formula if you prefer.

a) $\dfrac{7s+9}{(s-1)(s+3)} =$

b) $\dfrac{1}{(s-1)(s^2+1)} =$

c) $\dfrac{1}{s(s-1)} =$

3[16P]) Evaluate the following convolutions. You may use the Laplace transform if you prefer.

a) $t^2 * t =$

b) $t * e^{-4t} =$

3[36P]) Compute the inverse Laplace transform for each of the following functions:

a) $\mathcal{L}^{-1}\left(\dfrac{5}{s^2+6s+9}\right) =$

b) $\mathcal{L}^{-1}\left(\frac{2}{(s-1)(s^2+1)}\right) =$

c) $\mathcal{L}^{-1}\left(\frac{s}{s^2+4s+5}\right)(t) =$

d) $\mathcal{L}^{-1}\left(\frac{s+1}{((s+1)^2+1)^2}\right) =$

A short table of Laplace transforms and inverse Laplace transform

$$
\begin{aligned}
\mathcal{L}(af(t)+bg(t))(s) &= aF(s)+bG(s) \\
\mathcal{L}(e^{at}f(t))(s) &= F(s-a) \\
\mathcal{L}(-tf(t))(s) &= \frac{d}{ds}F(s) \\
\mathcal{L}(1)(s) &= \frac{1}{2} \\
\mathcal{L}(t^n)(s) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}(e^{at})(s) &= \frac{1}{s-a} \\
\mathcal{L}(\cos(bt))(s) &= \frac{s}{s^2+b^2} \\
\mathcal{L}(\sin(bt))(s) &= \frac{b}{s^2+b^2} \\
\mathcal{L}(f'(t))(s) &= sF(s)-f(0) \\
\mathcal{L}(f*g(t))(s) &= F(s)G(s) \\
\mathcal{L}^{-1}\left(\frac{1}{(s^2+1)^2}\right)(t) &= \frac{1}{2}(\sin(t)-t\cos(t)) \\
\mathcal{L}^{-1}\left(\frac{s}{(s^2+1)^2}\right)(t) &= \frac{1}{2}t\sin(t)
\end{aligned}
$$

MATH 2065-4, Fall 2010 Name: Solutions

Test 2, Thursday, Sept 26, 2010. For partial credit, show all your work!

1[24P]) Compute the Laplace transform of each of the following functions:

a) $\mathcal{L}\left(t^3e^{-2t}\right)(s) = \dfrac{6}{(s+2)^4}$

b) $\mathcal{L}\left(t\sin(4t) + e^{-3t}\cos(t)\right)(s) = \dfrac{8s}{(s^2+16)^2} + \dfrac{s+3}{(s+3)^2+1}$

c) $\mathcal{L}\left((2te^t+1)(t^3+2)\right)(s) = \dfrac{48}{(s-1)^5} + \dfrac{6}{s^4} + \dfrac{4}{(s-1)^2} + \dfrac{2}{s}$

2[24P]) Find the partial fraction decomposition of each of the following rational functions. You may use the convolution formula if you prefer.

a) $\dfrac{7s+9}{(s-1)(s+3)} = \dfrac{4}{s-1} + \dfrac{3}{s+3}$

Set

$$\frac{7s+9}{(s-1)(s+3)} = \frac{A}{s-1} + \frac{B}{s+3}.$$ Then

$$(A+B)s + 3A - B = 7s + 9$$ or

$$A+B=7$$
$$3A-B=9$$
$$4A=16$$ or $A = 16/4 = 4$

$$B = 7 - A = 7 - 4 = \underline{3}$$

b) $\frac{1}{(s-1)(s^2+1)} = \frac{1}{2}\left[\frac{1}{s-1} - \frac{s+1}{s^2+1}\right]$

write

$\frac{1}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+1}{s^2+1}$. Then $\begin{cases} A+B=0 \\ -B+C=0 \\ A-C=1 \end{cases}$

Thus $B=C$ and $\begin{matrix} A+B=0 \\ A-B=1 \end{matrix}$ or $A=1/2$, $B=C=-1/2$.

c) $\frac{1}{s(s-1)} = -\frac{1}{s} + \frac{1}{s-1}$

3[16P]) Evaluate the following convolutions. You may use the Laplace transform if you prefer.

a) $t^2 * t = \frac{1}{12}t^4$.

Use the Laplace transform: $\mathcal{L}(t^2 * t) = \frac{2}{s^3}\cdot\frac{1}{s^2} = \frac{2}{s^5}$

$= \frac{1}{12}\cdot\frac{4!}{s^5}$

b) $t * e^{-4t} = \frac{1}{4}t - \frac{1}{16} + \frac{1}{16}e^{-4t}$

$\int_0^t u e^{-4(t-u)}\,du = e^{-4t}\left[\int_0^t u e^{4u}\,du\right] = e^{-4t}\left[\frac{1}{4}ue^{4u}\Big|_0^t - \frac{1}{4}\int_0^t e^{4u}\,du\right] = e^{-4t}\left[\frac{1}{4}te^{4t} - \frac{1}{16}e^{4u}\Big|_0^t\right] = \frac{1}{4}t - \frac{1}{16} + \frac{1}{16}e^{-4t}$

3[36P]) Compute the inverse Laplace transform for each of the following functions:

a) $\mathcal{L}^{-1}\left(\frac{5}{s^2+6s+9}\right) = 5te^{-3t}$

b) $\mathcal{L}^{-1}\left(\frac{2}{(s-1)(s^2+1)}\right) = e^t - \cos t - \sin t$

$$\frac{2}{(s-1)(s^2+1)} = -\frac{s+1}{s^2+1} + \frac{1}{s-1} = \frac{-s}{s^2+1} - \frac{1}{s^2+1} + \frac{1}{s-1}$$

(use problem #2-b)

c) $\mathcal{L}^{-1}\left(\frac{s}{s^2+4s+5}\right)(t) = e^{-2t}\left[\cos t - \sin t\right]$

$$\frac{s}{s^2+4s+5} = \frac{s}{(s+2)^2+1} = \frac{s+2}{(s+2)^2+1} - \frac{2}{(s+2)^2+1}$$

d) $\mathcal{L}^{-1}\left(\frac{s+1}{((s+1)^2+1)^2}\right) = \frac{te^{-t}\sin t}{2}$

(use the last formula in the table)

A short table of Laplace transforms and inverse Laplace transform

$$
\begin{aligned}
\mathcal{L}(af(t)+bg(t))(s) &= aF(s)+bG(s) \\
\mathcal{L}(e^{at}f(t))(s) &= F(s-a) \\
\mathcal{L}(-tf(t))(s) &= \frac{d}{ds}F(s) \\
\mathcal{L}(1)(s) &= \frac{1}{2} \\
\mathcal{L}(t^n)(s) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}(e^{at})(s) &= \frac{1}{s-a} \\
\mathcal{L}(\cos(bt))(s) &= \frac{s}{s^2+b^2} \\
\mathcal{L}(\sin(bt))(s) &= \frac{b}{s^2+b^2} \\
\mathcal{L}(f'(t))(s) &= sF(s)-f(0) \\
\mathcal{L}(f*g(t))(s) &= F(s)G(s) \\
\mathcal{L}^{-1}\left(\frac{1}{(s^2+1)^2}\right)(t) &= \frac{1}{2}(\sin(t)-t\cos(t)) \\
\mathcal{L}^{-1}\left(\frac{s}{(s^2+1)^2}\right)(t) &= \frac{1}{2}t\sin(t)
\end{aligned}
$$

MATH 2065-4, Fall 2010 Name: Solutions

Test 3, Thursday, Nov. 18, 2010. For partial credit, show all your work!

1[12P]) For each of the following differential equations determine if it is linear (Y=linear) or not (N=not):

$L(y)$	Y	N
$y'' + y' + y = \sin(t)$	X	
$y'' + y'y = t$		X
$y'' + \cos(y) = \cos(t)$		X
$y'' + y' + \cos(t) = y$	X	

2[6P]) Show that $y(t) = t^{1/2}$ is a solution to the differential equation $4t^2y'' + y = 0$.

$y = t^{1/2}$

$y' = \frac{1}{2} t^{-1/2}$

$y'' = -\frac{1}{4} t^{-3/2}$

$4t^2 y'' + y = -t^{1/2} + t^{1/2} = 0$

3[7]) For the linear operator $L = D^2 + 4D + 4$ determine $L(te^{-t}) =$

$y = te^{-t}$

$y' = -te^{-t} + e^{-t}$

$y'' = te^{-t} - 2e^{-t}$

$L(te^{-t}) = te^{-t} - 2e^{-t} - 4te^{-t} + 4e^{-t} + 4te^{-t}$

$= te^{-t} + 2e^{-t}$

4[45P]) Find the general solution to each of the following differential equations:

a) $y'' - 3y' + 2y = e^t$. **Solution** $y(t) = c_1 e^t + c_2 e^{2t} - te^t$

$s^2 - 3s + 2 = (s-1)(s-2)$

$y_h = c_1 e^t + c_2 e^{2t}$

$y_p \Longleftrightarrow \frac{1}{(s-1)^2(s-2)} = \frac{-1}{(s-1)^2} + \frac{1}{(s-1)(s-2)}$

$y_p = -te^t$

b) $y'' - 4y' + 4y = e^t$. **Solution** $y(t) = c_1 e^{2t} + c_2 te^{2t} + e^t$

$s^2 - 4s + 4 = (s-2)^2$

$y_p = Ae^t,\ y_p' = Ae^t,\ y_p'' = Ae^t$

c) $y'' - 4y' + 5y = 1$. **Solution** $y(t) = e^{2t}(c_1 \cos(t) + c_2 \sin(t)) + \frac{1}{5}$

$s^2 - 4s + 5 = (s-2)^2 + 1$

5[15P]) Find the general solution to the differential equation $y'' + y' - 2y = te^t$.

Solution: $y(t) =$

$s^2 + s - 2 = (s+2)(s-1) \quad y_h(t) = c_1 e^{-2t} + c_2 e^t$

$y_p(t)$: write $y_p = c_1(t) e^{-2t} + c_2(t) e^t$. Then

$c_1' e^{-2t} + c_2' e^t = 0$

$-2c_1' e^{-2t} + c_2' e^t = te^t$

$3c_2' e^t = te^t$ or $c_2' = \frac{1}{3} t$

$c_2 = \frac{1}{3} \int t\,dt = \frac{1}{6} t^2$

$3c_1' e^{-2t} = -te^t$

$c_1' = -\frac{1}{3} t e^{3t}$

$c_1 = -\frac{1}{3} \int t e^{3t} dt$

$= -\frac{1}{9} t e^{3t} + \frac{1}{9} \int e^{3t} dt$

$= -\frac{1}{9} t e^{3t} + \frac{1}{27} e^{3t}$.

$y_p = -\frac{1}{9} t e^t + \frac{1}{27} e^t + \frac{1}{6} t^2 e^t$

$y = c_1 e^{-2t} + c_2 e^t + \left(\frac{1}{6} t^2 - \frac{1}{9} t + \frac{1}{27}\right) e^t$.

6[15P]) Solve the initial value problem $t^2 y'' + ty' - 4y = 0$, $y(1) = 1$, $y'(1) = 1$.

Solution: $y(t) = \frac{1}{4}(3t^2 - t^{-2})$

Euler equation: $Y''(x) - 4Y(x) = 0$, $Y(x) = c_1 e^{2x} + c_2 e^{-2x}$

$y(t) = c_1 t^2 + c_2 t^{-2}$

$y'(t) = 2c_1 t^2 - 2c_2 t^{-2}$

$c_1 + c_2 = 1$

$2c_1 - 2c_2 = 1$

MATH 2065-4, Fall 2010 Name: ____________________

Final, Tuesday, Dec. 7, 2010, 12:30-2:30

For partial credit, show all your work!

1[10P]) Let $L = D^2 + 3tD + 2$. What is $L(e^t + t) =$

2[48P]) Find the general solution to each of the following differential equation:

a) $y' = -2ty$. **Solution:** $y(t) =$

b) $y' = \dfrac{3y - t}{2ty}$. **Solution:** $y(t) =$

b) $y'' + y' - 6y = e^{2t}$. **Solution:** $y(t) =$

e) $y'' - 6y' + 10y = 0$. **Solution:** $y(t) =$

3[45P]) Solve each of the following initial value problems.

a) $y' - \frac{2}{t}y = t^2\cos(t)$, $y(\pi) = 3$. **Solution:** $y(t) =$

b) $y'' + y' - 2y = 1$, $y(0) = 0$, $y'(0) = 1$. **Solution** $y(t) =$

c) $y' - y = \begin{cases} 1 & \text{if } 0 \le t < 2 \\ -1 & \text{if } 1 \le t < \infty \end{cases}$ $\quad y(0) = 0$. **Solution:** $y(t) =$

4[8P]) Compute the Laplace transform $\mathcal{L}(t\cos(t))(s) =$

5[16P]) Compute the inverse Laplace transform for each of the following functions:

a) $\mathcal{L}^{-1}\left(\frac{7s+9}{s^2+2s-3}\right)(t) =$

b) $\mathcal{L}^{-1}\left(\frac{s}{(s-1)(s^2+1)}\right)(t) =$

5[8P]) Evaluate the convolution $t * e^t =$

6)[15P]) A tank contains 100 gal of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 2 gal/min, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t.

MATH 2065-4, Fall 2010 Name: ______________________

Final, Tuesday, Dec. 7, 2010, 12:30-2:30

For partial credit, show all your work!

1[10P]) Let $L = D^2 + 4tD + 1$. Show that $\sin(t)$ is a solution to the differential equation $L(y) = 4t\cos(t)$.

2[48P]) Find the general solution to each of the following differential equation:

a) $y' = -2\cos(t)y$. **Solution:** $y(t) =$

b) $y' = \dfrac{3y^2 - t^2}{2ty}$. **Solution:** $y(t) =$

b) $y'' - 3y' + 2y = e^t$. **Solution:** $y(t) =$

e) $y'' - 6y' + 10y = 0$. **Solution:** $y(t) =$

3[45P]) Solve each of the following initial value problems.

a) $y' - \frac{3}{t}y = t^3e^t$, $y(1) = 3$. **Solution:** $y(t) =$

b) $y'' + y' - 2y = 1$, $y(0) = 0$, $y'(0) = 1$. **Solution** $y(t) =$

c) $y' - y = \begin{cases} 1 & \text{if } 0 \leq t < 2 \\ -1 & \text{if } 1 \leq t < \infty \end{cases}$ $y(0) = 0$. **Solution:** $y(t) =$

4[8P]) Compute the Laplace transform $\mathcal{L}(t\cos(t))(s) =$

5[16P]) Compute the inverse Laplace transform for each of the following functions:

a) $\mathcal{L}^{-1}\left(\frac{7s+9}{s^2+2s-3}\right)(t) =$

b) $\mathcal{L}^{-1}\left(\frac{s}{(s-1)(s^2+1)}\right)(t) =$

5[8P]) Evaluate the convolution $t * e^t =$

6)[15P]) A tank contains 100 gal of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gal/min, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t.