Assignment 3, due Thursday, Nov. 5, 2009, **before class**
For partial credit, show all your work!

1[12P]) Which one of the following defines an inner product on the given vector space V. **If it is not an inner product list at least one condition from the definition that does not work and show why it does not work!**

a) $V = \mathbb{R}^2$, $((x, y), (u, v)) = xv$. Not an innerproduct.

$((1,0),(1,0)) = 0$ but $(1,0) \neq (0,0)$

b) $V = \mathbb{R}^3$, $((x, y, z), (u, v, w)) = 2xu + 4yv + zw$. Inner product

(All the number $2, 4, 1 > 0$, so this was done in class)

c) $V = \mathbb{R}^3$, $((x, y, z), (u, v, w)) = 3xu - 2yv + 10zw$. Not an inner product

$((0,1,0),(0,1,0)) = -2 < 0$.

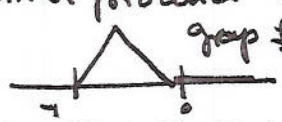
d) $V = C([-1, 1])$ (the space of continuous **real valued** functions on $[-1, 1]$), $(f, g) = \int_{-1}^{1} f(t)g(t)\, e^t\, dt$. Is an inner product (was shown in class).

① $(f,f) = \int_{-1}^{1} f(t)^2 e^t\, dt \geq 0$ (2) If $= 0$, then $f(t)^2 e^t = 0$, so $f(t) = 0$.

③ Fix g. Then $f \rightsquigarrow \int_{-1}^{1} f(t) g(t) e^t dt$ is linear because the integral is linear. (4) $(f,g) = \int_{-1}^{1} f(t)g(t)e^t dt = \int_{-1}^{1} g(t)f(t)e^t dt = (g,f)$.

e) $V = C([-1, 1])$, $\int_{-1}^{1} f(t)g(t)t\, dt$.

Not inner product because $t < 0$ on $[-1,1]$

Take , then $\int_{-1}^{1} f(t)^2 t\, dt < 0$.

g) $V = C([-1, 1])$, $(f, g) = \int_0^1 f(t)g(t)\, dt$.

Not inner product. Take f as in (e), then $f \neq 0$, but $(f,f) = 0$

2[6P]) Consider the inner product $((x,y,z),(u,v,w)) = 2xu + 5yv + 3zw$ on $\mathbb{R}^3$:

a) Evaluate $((1,-2,3),(1,1,-3)) =$ -35

Solution: $2+5(-2)\cdot 1+3\cdot 3\cdot(-3) = 2-10-27 = -35$

b) Are the vectors $(1,-1,1)$ and $(1,1,1)$ orthogonal with respect to this inner product? Yes

$((1,-1,1),(1,1,1)) = 2-5+3 = 0$

3[6P]) Evaluate the following inner product and norms using the inner product $(f,f) = \int_{-1}^{1} f(t)g(t)\,dt$ on $C([-1,1])$:

a) $(t^3+2t^2-t, t^2) =$ $4/5$

$(t^3+2t^2-t)\cdot t^2 = t^5+2t^4-t^3$. Thus $(f,g) = \int_{-1}^{1}(t^3+2t^2-t)t^2\,dt =$

$= \int_{-1}^{1} t^5+2t^4-t^3\,dt = \frac{t^6}{6} + \frac{2t^5}{5} - \frac{t^4}{4}\Big]_{-1}^{1} = \underline{\underline{4/5}}$

b) $\|t^3-t\| =$.

$\|t^3-t\|^2 = \int_{-1}^{1}(t^3-t)^2\,dt = \int_{-1}^{1} t^6-2t^4+t^2\,dt =$

$= \frac{t^7}{7} - \frac{2}{5}t^5 + \frac{1}{3}t^3\Big]_{-1}^{1}$

$= \frac{2}{7} - \frac{4}{5} + \frac{2}{3} = \frac{30-84+70}{105} = \frac{16}{105}$

4[6P]) Consider the inner product $((x,y,z),(u,v,w)) = x\bar{u} + y\bar{v} + z\bar{w}$ on $\mathbb{C}^3$.

a) Evaluate the inner product $((1+i, 2-3i, 3),(2i, 2+3i, 1)) =$ $-14i$

Solution: $((1+i, 2-3i, 3),(2i, 2+3i, 1)) =$

$= (1+i)\overline{(2i)} + (2-3i)\overline{(2+3i)} + 3\cdot 1 =$

$= -2(1+i)i + (2-3i)\cdot(2-3i) + 3 =$

$= -2i + 2 + 4 - 6i - 6i - 9 + 3$

$= \underline{\underline{-14i}}$

b) Evaluate the norm $\|(2+i, 3-2i, i)\| =.$ $\sqrt{19}$

$\|(2+i, 3-2i, i)\|^2 = |2+i|^2 + |3-2i|^2 + |i|^2$

$= 4 + 1 + 9 + 4 + 1 = 19$