MATH 2025 Fall 2009

Test 1, Thursday, Sept 24, 2009

For partial credit, show all your work!

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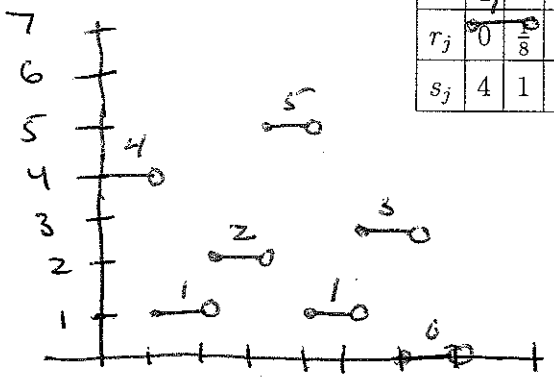
1 [7P]) Plot (3P) and write a formula (4P) using the functions $\varphi_j^{(\ell)}$ for the step function $\tilde{q}$ corresponding to the sample in the following table

j	0	1	2	3	4	5	6	7
r_j	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
s_j	4	1	2	5	1	3	0	7



$$4\varphi_0^{(3)} + \varphi_1^{(3)} + 2\varphi_2^{(3)} + 5\varphi_3^{(3)} + \varphi_4^{(3)} + 3\varphi_5^{(3)} + 7\varphi_7^{(3)}$$

2 [9P]) Calculate the *ordered Fast Haar Wavelet Transform* for the data $(0, 5, 1, 2)$.

Solution: $(2; 1/2; -5/2; -1/2)$

$$(0,5,1,2) \mapsto \left(\frac{5}{2}, \frac{3}{2}, -\frac{5}{2}, -\frac{1}{2}\right)$$

$$\mapsto \left(2; \frac{1}{2}; -\frac{5}{2}, -\frac{1}{2}\right)$$

3 [14P]) Calculate the *ordered Fast Haar Wavelet Transform* for the data $\mathrm{s} = (10, 10, 9, 3, 3, 7, 8, 6)$.

Solution $s^{(0)} = (7; 1; 2, -1; 0, 3, -2, 1)$

$s^{(3)} = (10, 10, 9, 3, 3, 7, 8, 6)$

$\mapsto s^{(2)} = (10; 6, 5, 7; 0, 3, -2, 1)$

$\mapsto s^{(1)} = (8, 6; 2, -1; 0, 3, -2, 1)$

$\mapsto s^{(0)} = (7; 1; 2, -1; 0, 3, -2, 1)$

4[9P]. Write the results in the **first** step in problem 3 as a combination of the functions φ and ψ.

$$10\varphi_0^{(3)} + 10\varphi_1^{(3)} + 9\varphi_2^{(3)} + 3\varphi_3^{(3)} + 3\varphi_4^{(3)} + 7\varphi_5^{(3)} + 8\varphi_6^{(3)} + 6\varphi_7^{(3)}$$

$$\rightarrow 10\varphi_0^{(2)} + 6\varphi_1^{(2)} + 5\varphi_2^{(2)} + 7\varphi_3^{(2)} + 3\psi_1^{(2)} - 2\psi_2^{(2)} + \psi_3^{(2)}$$

5[9P]) Assume that the *ordered Fast Haar Wavelet Transform* of a sample $s = (s_0, s_1, s_2, s_3)$ produces the results $s^{(2-2)} = (4; 1; 3, 2)$. Use the inverse Fast Haar Wavelet Transform to find s.

$S = S^{(2)} = (8, 2, 5, 1)$.

$(4; 1; 3, 2) \rightarrow (5, 3; 3, 2)$

$\rightarrow (8, 2, 5, 1)$

6[9P]) Find the *In-Place Haar Wavelet Transform* of the sample $s = (8, 6, 7, 3, 1, 1, 2, 4)$.

$S^{(0)} = (4, 1, 1, 2, 2, 0, -1, -1)$

$(8, 6, 7, 3, 1, 1, 2, 4)$

$\rightarrow (7, 1, \underline{5}, 2, \underline{1}, 0, \underline{3}, -1)$

$\rightarrow (\underline{\underline{6}}, 1, 1, 2, \underline{\underline{2}}, 0, -1, -1)$

$\rightarrow (\underline{\underline{\underline{4}}}, 1, 1, 2, 2, 0, -1, -1)$

7[14P]) Assume that the In-Place Haar Wavelet Transform of the sample $s = (s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ results in $s^{(3-2)} = (8, -1, 3, 1, 2, -1, 2, 0)$. Use the inverse In-Place Haar Wavelet transform to find s. (10, 12, 6, 4, 3, 5, 0, 0)

$s^{(3-2)} = (\underline{8}, -1, \underline{3}, 1, \underline{2}, -1, \underline{2}, 0) \rightarrow s^{(3-1)} = (\underline{11}, -1, \underline{5}, 1, \underline{4}, -1, \underline{0}, 0)$

$\rightarrow (10, 12, 6, 4, 3, 5, 0, 0)$

8[9P] Assume that the In-Place Haar Wavelet Transform of the sample $s = (s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7)$ results in $s^{(3-3)} = (9, 1, 3, 2, -2, 3, 2, 1)$.

a) What is the average of the sample that we started with? 9

b) Identify the value of $c_1^{(3-2)}$. 2

c) Explain how $c_0^{(3-1)} = 1$ relates to the sample. Average change $s_0 \Rightarrow s_1$

9[14P]) Evaluate the 2D Wavelet transform of $\begin{pmatrix} 9 & 7 & 6 & 2 \\ 5 & 3 & 4 & 4 \\ 8 & 2 & 4 & 0 \\ 6 & 0 & 2 & 2 \end{pmatrix}$.

$\begin{pmatrix} 4 & 1 & 1 & 1 \\ 1 & 0 & 3 & 1 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 9 & 7 & 6 & 2 \\ 5 & 3 & 4 & 4 \\ 8 & 2 & 4 & 0 \\ 6 & 0 & 2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 8 & 1 & 4 & 2 \\ 4 & 1 & 4 & 0 \\ 5 & 3 & 2 & 2 \\ 3 & 3 & 2 & 0 \end{pmatrix}$

$\rightarrow \left(\begin{array}{cc|cc} 6 & 1 & 4 & 1 \\ 2 & 0 & 0 & 1 \\ \hline 4 & 3 & 2 & 1 \\ 1 & 0 & 0 & 1 \end{array}\right) \mapsto \left(\begin{array}{cc|cc} 6 & 4 & 1 & 1 \\ 4 & 2 & 3 & 1 \\ \hline 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|cc} 5 & 1 & 1 & \\ 3 & 1 & & \\ \hline & & & \\ & & & \end{array}\right)$

$\rightarrow \left(\begin{array}{cc|c} 4 & 1 & \\ 1 & 0 & \\ \hline & & \end{array}\right)$

10[6P]) Assume that the 2D Wavelet Transform of $\begin{pmatrix} s_{0,0} & s_{0,1} \\ s_{1,0} & s_{1,1} \end{pmatrix}$ results in $\begin{pmatrix} 8 & -1 \\ 2 & 1 \end{pmatrix}$

a) What is the meaning of the number 8? Average of the sample

b) What is the meaning of the number 2? Average change going from row 1 to row 2