More homework

10.69

$f(x_1, x_2, x_3) = (x_1 x_2 \cos(x_3),\ x_2 \sin(x_3))$. Then

$$\frac{\partial(f_1, f_2)}{\partial(x_2, x_3)} = \det\begin{pmatrix} x_1 \cos(x_3) & \sin(x_3) \\ -x_1 x_2 \sin(x_3) & x_2 \cos(x_3) \end{pmatrix}$$

$$= x_1 x_2 \cos^2(x_3) - (-x_1 x_2) \sin^2(x_3)$$

$$= x_1 x_2 .$$

10.73 [See solutions to Test # 3]

10.77 $Df(x_0) = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \end{pmatrix}$, $x_2 = x_2(x_1)$, $x_3 = x_3(x_1)$.

Direct solution:

$$f(x_1, x_2(x_1), x_3(x_1)) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dx_1} + \frac{\partial f}{\partial x_3}\frac{dx_3}{dx_1} = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ \frac{dx_2}{dx_1} \\ \frac{dx_3}{dx_1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

which leads to

$$x_2' + 2x_3' = -1$$
$$3x_3' = 1$$

so $x_3' = 1/3$ and $x_2' = -1 - 2/3 = -5/3$

10.79

i) $\vec{f}(x_1, x_2, x_3) = \begin{pmatrix} x_2 + x_3 - 1 \\ x_1 - x_2^2 - x_3^2 \end{pmatrix} \in C^1(\mathbb{E}^3, \mathbb{E}^2)$

because the components are polynomials.

ii) $\dfrac{\partial(f_1, f_2)}{\partial(x_2, x_3)} = \det\begin{pmatrix} 1 & 1 \\ -2x_2 & -2x_3 \end{pmatrix} = -2(x_2 - x_3) \neq 0$

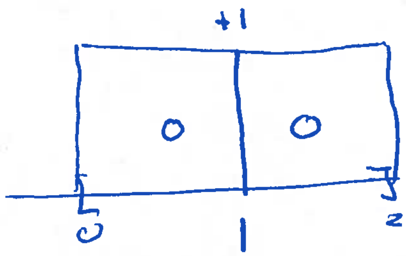
for $x_2 \neq x_3$.

iii) $$\begin{pmatrix} x_2' \\ x_3' \end{pmatrix} = -\left(\frac{d\bar{f}}{d\bar{y}}\right)^{-1} \begin{pmatrix} \frac{\partial f_1}{\partial x_1} \\ \frac{\partial f_2}{\partial x_1} \end{pmatrix}$$

$$= -\begin{pmatrix} \frac{x_3}{x_2 - x_3} & \frac{1}{2(x_2-x_3)} \\ \frac{x_2}{x_2 - x_3} & \frac{-1}{2(x_2-x_3)} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2(x_2-x_3)} \\ \frac{-1}{2(x_2-x_3)} \end{pmatrix} = \frac{1}{2(x_2-x_3)} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

11.6)



Let $P = \{[0, 1-\frac{1}{n}]\times[0,1], [1-\frac{1}{n}, 1+\frac{1}{n}]\times[0,1], [1+\frac{1}{n}, 2]\times[0,1]\}$

Then

$U(f,P) = 0 + 1\cdot\frac{2}{n} + 0 = \frac{2}{n}$

$L(f,P) = 0$

Take $n > 16$.

11.7) Obvious.

11.8) If $P = \{B_j\}$ is a partition, then

then

$$M_i(f^-) = \begin{cases} -m_i(f) & \text{if } f|B_i \leq 0 \\ 0 & \text{if } f|B_i \geq 0 \end{cases}$$

$$m_i(f^-) = \begin{cases} -M_i(f) & \text{if } f|B_i \leq 0 \\ 0 & \text{if } f|B_i \geq 0 \end{cases}$$

Thus

$$(M_i(f^-) - m_i(f^-))\mu(B_i) = \begin{cases} (M_i(f) - m_i(f))\mu(B_i) & \text{if } f|B_i \leq 0 \\ 0 & \text{if } f|B_i > 0 \end{cases}$$

$$\leq (M_i(f) - m_i(f))\mu(B_i)$$

It follows that

$$0 \leq U(f^-, P) - L(f^-, P) \leq U(f, P) - U(f, P)$$

Hence $f^- \in \mathcal{R}[a,b]$ by Thm 11.1.3.

As

$f^+ = f + f^-$ and $\mathcal{R}[a,b]$ is a vector space

it follows that $f^+ \in \mathcal{R}[a,b]$.

<u>11.9)</u> $|f| = f^+ + f^-$ and $f^+, f^- \in \mathcal{R}[a,b]$. As $\mathcal{R}[a,b]$ is a vector space it follows that $|f| \in \mathcal{R}[a,b]$.

<u>11.10)</u> $f = 1_{\mathbb{Q} \cap [0,1]} - 1_{[0,1] \setminus \mathbb{Q} \cap [0,1]} \notin \mathcal{R}[0,1]$

but $|f| = 1_{[0,1]} \in \mathcal{R}[0,1]$.