

Math 2025, Homework #2, Due Tuesday, Nov. 4

Name: _____

Recall the inner products:

1) On \mathbb{F}^n : $\langle u, v \rangle = u_1 \overline{v_1} + \dots + u_n \overline{v_n}$.

2) If V is the space of piecewise continuous function on $[0, 1[$ then $\langle f, g \rangle = \int_0^1 f(t) \overline{g(t)} dt$.

Recall also that two vectors are said to be **orthogonal** if $\langle u, v \rangle = 0$.

The **norm** of a vector is the real number $\|u\| = \sqrt{\langle u, u \rangle}$.

Evaluate the following inner products:

1) $\langle (1, 3, 4), (3, -1, 4) \rangle =$

2) $\langle (i, 1 + i, 3), (i, 2 - i, 2) \rangle =$

3) $f(t) = t$ and $g(t) = e^t$.

4) $f(t) = \varphi_1^2(t)$ and $g(t) = \varphi_3^2(t)$. $\langle f, g \rangle =$

5) $f(t) = \varphi(t)$ and $g(t) = \psi(t)$. $\langle f, g \rangle =$

Evaluate the norm of the following vectors:

6) $\mathbf{u} = (1, 2, -2)$. $\|\mathbf{u}\| =$

7) $\mathbf{u} = (1, i)$. $\|\mathbf{u}\| =$

8) $f(t) = t + it^2$. $\|f\| =$

9) $f(t) = \psi(t)$. $\|f\| =$

Are the following vectors orthogonal or not?

10) $\mathbf{u} = (1, -1, 2)$ and $\mathbf{v} = (1, 1, 0)$.

11) $f(t) = \cos(2\pi t)$ and $g(t) = \sin(2\pi t)$.

12) $f(t) = \varphi_0^2(t)$ and $g(t) = \psi_0^2(t)$.

13) $f(t) = t$ and $g(t) = t^2$.

14) $f(t) = t$ and $g(t) = 3t - 4t^2$.