Math 2025, Homework 3, due Thursday, Nov 14, 9.10 AM

Name: _____

Consider the vector space V of real valued piecewise continuous functions on $[0,1)$ with the inner product $< f,g >= \int_0^1 f(t)g(t)\,dt$. Let $\varphi_j^1(t) = \varphi(2t-j)$ as usually. Let $f(t) = t^2$, $g(t) = t$, and $h(t) = 2t - 3t^2$.

1) Evaluate the inner products:

a) $< f,g >=$

b) $< g,\varphi_0^1 >=$

c) $< g,\varphi_1^1 >=$

d) $< f,h >=$

e) $< h,\varphi_1^1 >=$

2) Find the norm of the following functions:

a) $\|f\| =$

b) $\|g\| =$

c) $\|\varphi_0^1\| =$

d) $\|\varphi_1^1\| =$

3) Consider the subspace $V_1 = \{s_0\varphi_0^1 + s_1\varphi_1^1 \in V \mid s_0, s_1 \in \mathbb{R}\}$. Let

$$p = \frac{< g,\varphi_0^1 >}{\|\varphi_0^1\|^2}\varphi_0^1 + \frac{< g,\varphi_1^1 >}{\|\varphi_1^1\|^2}\varphi_1^1 \ .$$

a) Draw the graph of p and g.

b) Evaluate $\|g-p\|^2 =$

c) Show that if $q \in V_1$ then $g - p \perp q$.