

Math 2025, Section I

Wavelets made easy

Textbook: We will use lecture notes written for this class. Please go to

www.math.lsu.edu/~olafsson/teaching.html

to download the notes and other material. We also partially follow the book *Wavelets made easy*, author Yves Nievergelt.

Time: 9:10–10:30, Tuesday and Thursday in Lockett 285)

Instructor: Gestur Olafsson

Office: 322 Lockett

Office Hours: T, Th 10:40–11:20. You can also contact me by e-mail, olafsson@math.lsu.edu, for other appointments.

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web-page: www.math.lsu.edu/~olafsson. This syllabus, lecture notes, homework problems, test dates, and solutions to tests, quizzes and homeworks will be available on this web-page. You can also find old quizzes and tests, with solution, here.

SYLLABUS, according to the book *Wavelets made easy*

- Vector spaces. This is Chapter 4 in the book:
 - (1) Vector spaces;
 - (2) Subspaces;
 - (3) Examples, spaces of functions;
 - (4) Linear maps;
 - (5) Inner products and norms;
 - (6) Generating sets, linear independence, basis
 - (7) The Gram-Schmidt orthogonalization process;
 - (8) Orthogonal Projections
- Haar's Simple Wavelets, Chapter 1
- Multidimensional wavelets and applications. This is Chapter 2;
- Selected material from section 3;
- Material on the Discrete Fourier Transform and the Fast Fourier Transform from chapter 5;
- If there is time, then we will discuss some material from chapter 6 on Fourier series

There will be a presentation in November showing several applications of the Wavelet transform. The MatLab code for those presentations can be downloaded from the web-page www.math.lsu.edu/~olafsson/teaching.html.

The theory of wavelets is a relatively recent mathematical theory. It is the basic theory behind several modern applications in storage of electronic information, data compression, image reconstruction and electronic transmission of information. The applications includes the storage of finger prints (see: <http://www.c3.lanl.gov/brislawn/FBI/FBI.html>), and the new jpg-standard.

The basic ideas can be formulated using the language of **linear algebra**: Vector spaces, subspaces, linear maps, inner product, orthogonal projections, and basis. Related related concept in analysis are: Vector spaces of functions, approximation of functions using basic functions (in our case wavelets), dilation and translation, change of basis.

- We start by discussing some **linear algebra** from Chapter 4. The important concepts are as mentioned above: Vector spaces, linear maps, inner product, basis and orthogonal basis, orthogonal projections. We will give several examples, that will be used later.
- Then we discuss the simplest wavelet, the Haar wavelet in one dimension. The main topic is the **fast Haar wavelet transform**. This is a special case of orthogonal projections and change of basis.
- After that we discuss the **two dimensional Haar wavelet**, general construction of wavelets, and some applications.
- The wavelet transform is only one example of *integral transforms*. The **Fourier Transform** is much older. It has become an indispensable tool in mathematics and applied sciences. We will discuss some aspects of the Fourier Transform starting with the Fast Fourier transform. Before doing that, we will need to introduce the field of complex numbers, and the complex exponential function. If there is enough time, then we will also go over the the **Discrete Fourier Transform/Fourier series**

GRADINGS

- There will be **three** tests in class (each 100 points)
 - Tuesday, September 21;
 - Thursday, October 21;
 - Tuesday, November 23
- There will be quizzes in class or home work **every week**, 7 highest scores will be counted towards to final grade (70 points). **There are no make-up quizzes except you contact me before class.**
- The final exam (200 points) will take place:
 - **Thursday, Dec. 9, 7:30–9:30, in room Lockett 285.**

Points

Tests during the semester	300
Homework/Quizzes	70
Final	200
Total	570

Final Grades

A > 513, B > 456, C > 399, D ≥ 342. F < 342