Math 2058-1, Test # 2. Fall 2007 **Name:** ______________________

1[10P]) Find the maximum and minimum value of the function $f(x,y) = x^2y^2$ on $D = \{(x,y) \mid x^2+y^2 \le 1\}$.

The maximum is: $1/4$ The minimum is: 0

$\nabla f = 2xy(y,x) = (0,0)$ only if $x=0$ or $y=0$. But then $f(0,y) = f(x,0) = 0$
As $f(x,y) \geq 0$ it follows that 0 is the minimum value. There are no further critical points inside D. On the boundary we have
$y^2 = 1-x^2$ or $f = x^2(1-x^2) = x^2 - x^4 = g(x)$.
$g'(x) = 2x - 4x^3 = 2x(1-2x^2) = 0$ if $x = 0$ or $x = \frac{\pm 1}{\sqrt{2}}$.
$f(0, \pm 1) = 0$ and $f(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}) = \frac{1}{4}$. At the end points
$x = \pm 1$ we have $f = 0$ so $\frac{1}{4}$ is the max

2[10P]) Find the point on the plain $4x+2y-2z = 22$ closest to the point $P(1,-1,2)$.
The point is: $(5,1,0)$

$f(x,y,z) = (x-1)^2 + (y+1)^2 + (z-2)^2$ is the function to minimize, with the constrain $g = 4x+2y-2z = 22$. Thus

$2(x-1) = \lambda \cdot 4$
$2(y+1) = 2\lambda$
$2(z-2) = -2\lambda$
$x = 2\lambda + 1$
$y = \lambda - 1$
$z = -\lambda + 2$

Insert this into g:
$8\lambda + \cancel{4} + 2\lambda - 2 + 2\lambda - 4 = 12\lambda - 2 = 22$
or $\lambda = 2$. Thus
$x = 5$ $(=5)$
$y = 1$
$z = 0$

3[12P]) Use Lagrange multipliers to find the maximum and the minimum of the function $x+2y$ subject to the constrains $x+y+z = 1$, $y^2+z^2 = 8$. The maximum is: 5 The minimum is: -3

Use $\nabla f = \lambda \nabla g + \mu \nabla h$
$1 = \lambda$
$2 = \lambda + 2y\mu = 1 + 2\mu y$
$0 = \lambda + 2z\mu = 1 + 2\mu z$
Thus:
$2\mu y = 1$
$2\mu z = -1 = -2\mu y$

we get (as $\mu \neq 0$)
$z = -y$
Inserting into g:
$x + 0 = 1$
Inserting into h
$2y^2 = 8$ or $y = \pm 2$.
$f(1,2,-2) = 5$, $f(1,-2,+2) = -3$

4[48P]) Evaluate the iterated integrals:

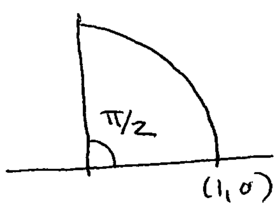
a) $\int_0^1 \int_0^1 x + xy + 3y \, dx dy =$ $\underline{9/4}$

$$\int_0^1 x+xy+3y\,dx = \frac{1}{2}x^2+\frac{1}{2}x^2y+3yx\Big]_0^1 = \frac{1}{2}+\frac{7}{2}y$$

$$\int_0^1 \frac{1}{2}+\frac{7}{2}y\,dy = \frac{1}{2}y+\frac{7}{4}y^2\Big]_0^1 = \frac{1}{2}+\frac{7}{4}=\frac{9}{4}$$

b) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy dx =$ $\underline{\frac{\pi}{4}(e-1)}$

use polar coordinates.

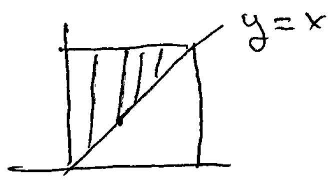


$$\int_0^{\pi/2}\int_0^1 e^{r^2} r\,dr\,d\theta = \frac{\pi}{2}\cdot\int_0^1 e^{r^2} r\,dr$$

$u=r^2$

$$= \frac{\pi}{4}\int_0^1 e^u\,du = \frac{\pi}{4}(e-1)$$

c) $\int_0^1 \int_x^1 \sin(y^2) \, dy dx =$ $\underline{\frac{1-\cos(1)}{2}}$

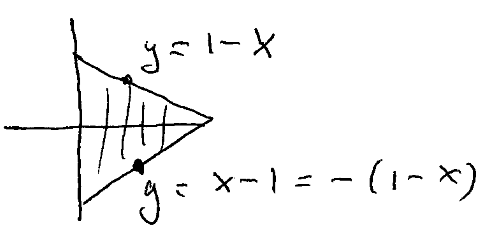


$$\int_0^1\int_0^y \sin(y^2)\,dx\,dy = \int_0^1 y\sin(y^2)\,dy$$

$u=y^2$

$$= \frac{1}{2}\int_0^1 \sin(u)\,du = \frac{1}{2}(-\cos u\Big]_0^1 = \frac{1-\cos(1)}{2}$$

d) $\iint_D xy \, dA =$ $\underline{0}$ where D is the triangular region with vertices $(0,1)$, $(1,0)$, $(0,-1)$.



$y \mapsto xy$ is odd. Thus

$$\int_{-(1-x)}^{1-x} xy\,dy = 0$$

f) $\iint_D \frac{1}{x^2+y^2}\,dA = \underline{2\pi \ln \frac{7}{2}}$ where D is the region $4 \le x^2+y^2 \le 49$.

use polar coordinates

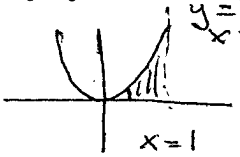
$$\int_0^{2\pi}\int_2^{7} \frac{1}{r^2}\, r\,dr\,d\theta = 2\pi \ln(r)\Big]_2^{7} = 2\pi \ln\left(\frac{7}{2}\right)$$

5[10P]) In the following integrals interchange the order of integration:

a) $\int_0^1 \int_0^{x^2} f(x,y)\,dydx = \int_0^1 \int_{\sqrt{y}}^{1} f(x,y)\,dxdy.$

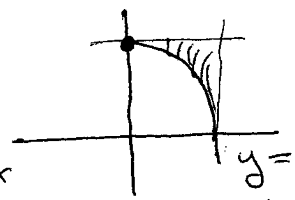
$y = x^2$

$x = \sqrt{y}$



$x = 1$

b) $\int_0^1 \int_{1-x^2}^{1} f(x,y)\,dydx = \int_0^1 \int_{\sqrt{1-y}}^{1} f(x,y)\,dxdy$



or,

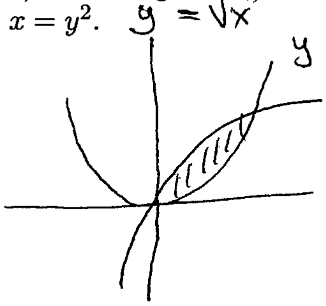
$$\int_0^1 \int_{1-y^2}^{1} f(x,y)\,dxdy = \int_0^1 \int_{\sqrt{1-x}}^{1} f(x,y)\,dydx$$

$y = 1-x^2$

$x = \sqrt{1-y}$

6[10]) Set up the following integrals. *You do not have to evaluate the integrals!*

a) The integral for the volume of the solid under the parabola x^2+y^2 and above the region $y = x^2$ and $x = y^2$. $y = \sqrt{x}$



$y = x^2$

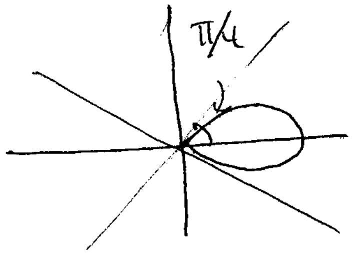
$y = \sqrt{x}$

$x^2 = \sqrt{x}; \quad x^4 = x, \quad x^3(x^3-1) = 0$

$x = 0$ or $x = 1$

$$\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 + y^2\,dy$$

b) The area enclosed by one loop of the four-leaved rose $r = \cos(2\theta)$.



$$\int_{-\frac{\pi}{4}}^{\pi/4} \int_0^{\cos(2\theta)} r\,dr\,d\theta$$