Math 7311, Analysis 1, Homework #5. Due Monday, Sept 24, at 11:30 in Class

1) Let $(X, \mathcal{A})$ be a measurable space and Y a non-empty set. If $f : X \to Y$ is a function define $\mathcal{B} = \{A \subseteq Y \mid f^{-1}(A) \in \mathcal{A}\}$. Show that $\mathcal{B}$ is a σ-algebra.

2) (Exercise 3.27, p. 52). For $\epsilon > 0$ construct a open dense subset $U_\epsilon \subset \mathbb{R}^n$ such that $\lambda(U_\epsilon) < \epsilon$ where λ is the Lebesgue measure on $\mathbb{R}^n$. (Think about $\mathbb{Q}^n \subset \mathbb{R}^n$.)

3) Let $(X, \mathcal{A})$ be a measurable space such that $\mathcal{A} \neq \mathcal{P}(X)$. Construct a function $f : X \to \mathbb{R}$ which is not measurable but $|f|$ is measurable.

4) (Compare to Bartle: The Elements of Integration and Lebesgue Measure, p. 26.)
a) Let $E \subset \mathbb{R}$ be open. Then $\lambda(E) = 0$ if and only if $E = \emptyset$.
b) If $K \subset \mathbb{R}$ is compact, then $\lambda(K) < \infty$.