Material for Test # 2

- You need all the material from Chapter 5, even if there will be no problem from Section 5.1. You will need to know Theorem 5.1.2.
- Work on the problems from **Solutions to problems from Chapter 5** on my webpage. The problems will be taken from this list. Some power series or functions might be different from those in the corresponding problem.
- There will be at **least** two problems from the home works.
- Note that we did not do the material on p. 127-129, so that will not be on the test.
- There will be problems where you have to find the radius of convergence and you will have to test the convergence at the endpoints.
- You will have to know the definition of the space ℓ_1 and ℓ_∞.
- You will need the following:
  1. Consider the power series $\sum_{k=0}^{\infty} c_k x^k$ and assume that $\lim_{k\to\infty} |c_{k+1}|/|c_k| = L$ exists. Then $R = 1/L$.
  2. Consider the power series $\sum_{k=0}^{\infty} c_k x^k$ and assume that $\lim_{k\to\infty} \sqrt[k]{|c_k|} = L$ exists. Then $R = 1/L$.
- You need to know the definition of a Banach space and its dual (Section 5.4).
- How to integrate and differentiate power series.
- If $f(x) = \sum_{k=0}^{\infty} c_k (x-a)^k$. Then $c_k = f^{(k)}(a)/k!$. You have to be able to use this to find c_k and/or to find $f^{(k)}(a)$ (given the coefficients c_k).
- Make sure you know how to prove:
  1. Problem 5.4.3;
  2. Theorem 5.5.1 part (a) and part (c).
  3. Theorem 5.5.2, Weierstrass M-test;
  4. The following part of Theorem 5.6.1: If the power series $\sum_{k=0}^{\infty} c_k x^k$ converges on $(-R, R)$, then $\sum_{k=0}^{\infty} c_k x^k$ converges uniformly on on closed sub-intervals $[\alpha, \beta] \subset (-R, R)$;
  5. Problem 5.8-3.