2

Math 1550-22, Test #~~1~~. Fall 2008 Name: ______________________________

For Partial Credit, show your Work

1[42P]) Calculate the derivatives:

a) $\frac{d}{dx}\sin\left(\frac{1}{x^2+1}\right) =$ ______________________

b) $\frac{d}{dx}\sqrt{6x+\sqrt{4x}} =$ __________________

c) $\frac{d}{dx}\left(\frac{x(x+2)}{(4x^2+1)(2x+2)}\right)$ ______________________

d) $\frac{d}{dx}\sin^{-1}(x^2+x-1) =$ ______________________

e) $\dfrac{d}{dx}8^{x^2-x} =$ ______________________

2[8P]) Let $h(x) = \sqrt{x}$. Find $h''(1) =$ ______________.

3[8P] Find the equation of the tangent line of $x^2y + 2xy = x + 2y$ at the point $(1, 1)$.

Answer: The equation of the tangent line is ______________________________.

4[9P]) A conical tank has height 3 m and radius 2 m at the top. Water flows in at a rate of 2 m^3/min. How fast is the water level rising when it is 2 m? (The volume of conical tank is $V = \dfrac{4}{3}\pi r^2 h$

Answer: The water level is rising ________________

5[8P]) Estimate the quantity $\sqrt{26} - 5 \approx$ ______________ using the *Linear Approximation*. Show your work, calculator will give you the wrong answer!

Math 1550-22, Test # 3. Fall 2008 Name: ______________________

For Partial Credit, show your Work. You may use that $\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$.

1[30P]) Suppose that

$$f(x) = \frac{1}{x} + \frac{1}{x-1}.$$

Then

$$f'(x) = -\frac{x^2 + (x-1)^2}{x^2(x-1)^2} \qquad \text{and} \qquad f''(x) = \frac{2(2x-1)(x^2 - x + 1)}{x^3(x-1)^3}.$$

(A) Find all critical values of $f(x)$. If there are no critical values, enter *None* . If there are more than one, enter them separated by commas.
Critical value(s) = ______________

(B) Use **interval notation** to indicate where $f(x)$ is increasing. If it is increasing on more than one interval, enter the union of all intervals where $f(x)$ is increasing.
Increasing: ______________

(C) Use **interval notation** to indicate where $f(x)$ is decreasing. If it is decreasing on more than one interval, enter the union of all intervals where $f(x)$ is decreasing.
Decreasing: ______________

(D) Find the x-coordinates of all local maxima of $f(x)$. If there are no local maxima, enter *None* . If there are more than one, enter them separated by commas.
Local maxima at x = ______________

(E) Find the x-coordinates of all local minima of $f(x)$. If there are no local minima, enter *None* . If there are more than one, enter them separated by commas.
Local minima at $x =$ ____________

(F) Use **interval notation** to indicate where $f(x)$ is concave up.
Concave up: ____________

(G) Use **interval notation** to indicate where $f(x)$ is concave down.
Concave down: ____________

(H) Find all inflection points of f. If there are no inflection points, enter *None* . If there are more than one, enter them separated by commas.
Inflection point(s) at $x =$ ____________

(I) Find all horizontal asymptotes of f. If there are no horizontal asymptotes, enter *None* . If there are more than one, enter them separated by commas.
Horizontal asymptote(s): $y =$ ____________

(J) Find all vertical asymptotes of f. If there are no vertical asymptotes, enter *None* . If there are more than one, enter them separated by commas.
Vertical asymptote(s): $x =$ ____________

(K) Use all of the preceding information to sketch a graph of f.

2[15P]) A landscape architect wished to enclose a rectangular garden on one side by a brick wall costing \$60/ft and on the other three sides by a metal fence costing \$10/ft. If the area of the garden is 42 square feet, find the dimensions of the garden that minimize the cost.
Length of side with bricks $x =$ ____________

Length of adjacent side $y =$ ____________

3[15P]) Use L'Hopital's Rule to evaluate the following limits:

a) $\lim_{x \to 0} \dfrac{1 - \cos(2x)}{\sin(3x)} =$ ________

b) $\lim_{x \to 0^+} \sqrt{x}\ln(x) =$ ____________

c) $\lim_{x \to \infty} \sqrt{x^2 + 3x + 2} - x =$ ____________

4[15P]) Use Newton's Method with the function $f(x) = x^2 - 2$ and initial value $x_0 = 1$ to calculate x_1 and x_2.

$x_1 =$ ________________ $x_2 =$ ________________

5[10P]) Evaluate the following two antiderivatives:

a) $\int x(x + 2x^3)\, dx =$ ____

b) $\int e^{3x-1}\,dx =$ ______________________

6[15P]) Let $f(x) = 2x^2 + x$.

a) Calculate R_4 on $[0,1]$. $R_4 =$ __________

b) For N an integer calculate R_N on $[0,1]$. $R_N =$ ____________________

b) Use (b) to find the are below the graph of $y = x^2$ and above the interval $[0,1]$.

The are is: __________

Math 1550-22, Test # 3. Fall 2008 Name: Solutions

For Partial Credit, show your Work. You may use that $\sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}$**.**

1[30P]) Suppose that

$$f(x) = \frac{1}{x} + \frac{1}{x-1}.$$

Then

$$f'(x) = -\frac{x^2 + (x-1)^2}{x^2(x-1)^2} \qquad \text{and} \qquad f''(x) = \frac{2(2x-1)(x^2-x+1)}{x^3(x-1)^3}.$$

(A) Find all critical values of $f(x)$. If there are no critical values, enter *None* . If there are more than one, enter them separated by commas.

Critical value(s) = None

$x^2 + (x-1)^2 > 0$ for all x where defined

(B) Use **interval notation** to indicate where $f(x)$ is increasing. If it is increasing on more than one interval, enter the union of all intervals where $f(x)$ is increasing.

Increasing: Not increasing

$f'(x) < 0$ for all x where defined

(C) Use **interval notation** to indicate where $f(x)$ is decreasing. If it is decreasing on more than one interval, enter the union of all intervals where $f(x)$ is decreasing.

Decreasing: $(-\infty) \cup (0,1) \cup (1,\infty)$

(D) Find the x-coordinates of all local maxima of $f(x)$. If there are no local maxima, enter *None* . If there are more than one, enter them separated by commas.

Local maxima at x = None

No local max or min as $f'(x)$ is never zero

(E) Find the x-coordinates of all local minima of $f(x)$. If there are no local minima, enter *None* . If there are more than one, enter them separated by commas.
Local minima at $x =$ None

(F) Use **interval notation** to indicate where $f(x)$ is concave up.
Concave up: $(0, 1/2) \cup (1, \infty)$

	$2x-1$	x^2-x+1	x^3	$(x-1)^3$	
posit.	$x > 1/2$	$\mathbb{R}$	$x > 0$	$x > 1$	$(0, \frac{1}{2}) \cup (1, \infty)$
negat	$x < \frac{1}{2}$	never	$x < 0$	$x < 1$	$(-\infty, 0) \cup (\frac{1}{2}, 0)$

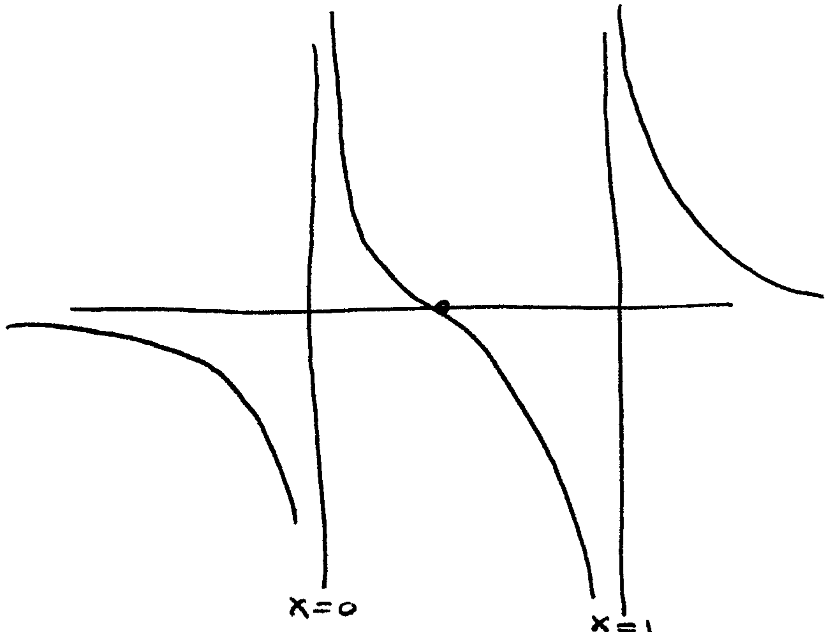
(G) Use **interval notation** to indicate where $f(x)$ is concave down.
Concave down: $(-\infty, 0) \cup (\frac{1}{2}, 1)$

(H) Find all inflection points of f. If there are no inflection points, enter *None* . If there are more than one, enter them separated by commas.
Inflection point(s) at $x =$ $1/2$

(I) Find all horizontal asymptotes of f. If there are no horizontal asymptotes, enter *None* . If there are more than one, enter them separated by commas.
Horizontal asymptote(s): $y =$ 0

(J) Find all vertical asymptotes of f. If there are no vertical asymptotes, enter *None* . If there are more than one, enter them separated by commas.
Vertical asymptote(s): $x =$ $0, 1$

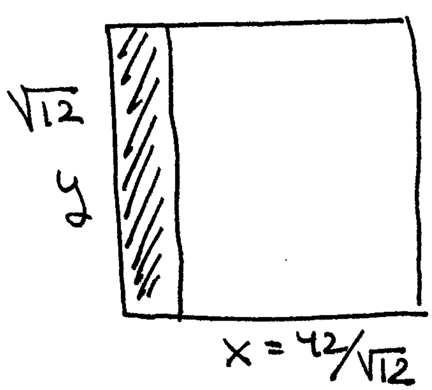
(K) Use all of the preceding information to sketch a graph of f.



2[15P]) A landscape architect wished to enclose a rectangular garden on one side by a brick wall costing \$60/ft and on the other three sides by a metal fence costing \$10/ft. If the area of the garden is 42 square feet, find the dimensions of the garden that minimize the cost.

Length of side with bricks y = $\sqrt{12}$

Length of adjacent side x = $42/\sqrt{12}$



Area $= xy = 42$, $x = \frac{42}{y}$

cost $= 60y + 20x + 10y$

$= 70y + \frac{840}{y}$

cost$' = 70 - \frac{840}{y^2} = 0$, $y^2 = \frac{840}{70} = 12$

$y = \sqrt{12}$

$x = 42/\sqrt{12}$

3[15P]) Use L'Hopital's Rule to evaluate the following limits:

a) $\lim_{x\to 0} \frac{1-\cos(2x)}{\sin(3x)} =$ 0

$\|$

$\lim_{x\to 0} \frac{2\sin(2x)}{3\cos(3x)} = 0$

b) $\lim_{x\to 0^+} \sqrt{x}\ln(x) =$ $\underline{\quad 0 \quad}$

$$\lim_{x\to 0^+} \sqrt{x}\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1/2}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = \lim_{x\to 0^+} \frac{-2x^{3/2}}{x} =$$

$$= \lim_{x\to 0^+} (-2\sqrt{x}) = 0$$

c) $\lim_{x\to\infty} \sqrt{x^2+3x+2} - x =$ $\underline{\quad 3/2 \quad}$

$$\| $$

$$\lim_{x\to\infty} x\left(\sqrt{1+\frac{3}{x}+\frac{2}{x^2}} - 1\right) = \lim_{x\to\infty} \frac{\sqrt{1+\frac{3}{x}+\frac{2}{x^2}} - 1}{1/x}$$

$u = 1/x$

$$= \lim_{u\to 0} \frac{\sqrt{1+3u+2u^2} - 1}{u} = \lim_{u\to 0} \frac{\frac{3+4u}{2\sqrt{1+3u+2u^2}}}{1} = 3/2$$

4[**15P**]) Use Newton's Method with the function $f(x) = x^2 - 2$ and initial value $x_0 = 1$ to calculate x_1 and x_2. $x_1 =$ $\underline{\quad 3/2 \quad}$ $x_2 =$ ______

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{1}{2}x_n + \frac{1}{x_n}$$

$$x_0 = 1: \quad x_1 = \frac{1}{2} + 1 = 3/2$$

$$x_2 = \frac{3}{4} + \frac{2}{3} = \frac{9+8}{12} = \frac{17}{12}$$

5[**10P**]) Evaluate the following two antiderivatives:

a) $\int x(x+2x^3)\,dx =$ $\underline{\quad \frac{1}{3}x^3 + \frac{2}{5}x^5 + C \quad}$

$$\| \int x^2 + 2x^4\,dx = \frac{1}{3}x^3 + \frac{2}{5}x^5 + C$$

Math 1550-22, Test # 4. Fall 2008 Name: Solutions

For Partial Credit, show your Work.

1[14P]) Evaluate the integrals:

a) $\int_{-2}^{2} (1 + t^2 - t^3)\, dt =$ $\underline{28/3}$

1 and t^2 are even, t^3 odd. The integral is therefore the same as

$$2 \int_0^2 1 + t^2\, dt = 2\left[t + t^3/3\right]_0^2 = 2\left[2 + \frac{8}{3}\right] = \frac{28}{3}$$

b) $\int_0^{\pi/4} \tan^2(x) \sec^2(x)\, dx =$ $\underline{1/2}$

set $u = \tan(u)$. Then $du = \sec^2(x)dx$

$$\int_0^1 u\, du = \frac{u^2}{2}\Big]_0^1 = 1/2$$

2[7P]) Calculate the derivative $\frac{d}{dx}\int_0^{x^2} \sqrt{t}\, dt =$ $\underline{2x\sqrt{x^2}}$ $= 2x|x|.$

Use the chain rule and the fundamental theorem of calculus

3[21P]) Evaluate the integrals:

a) $\int x\sqrt{1+x^2}\, dx =$ $\underline{\frac{1}{3}(1+x^2)^{3/2}}$ $+ C$

set $u = 1 + x^2$, $du = 2x dx$, $x dx = \frac{1}{2} du$

$$\frac{1}{2}\int u^{1/2} du = \frac{1}{2}\cdot\frac{2}{3} u^{3/2} + C = \frac{1}{3}(1+x^2)^{3/2} + C$$

b) $\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}(1+x^2)^{5/2} - \frac{1}{3}(1+x^2)^{3/2} + C$

$u = 1+x^2;\quad x^2 = u-1:$

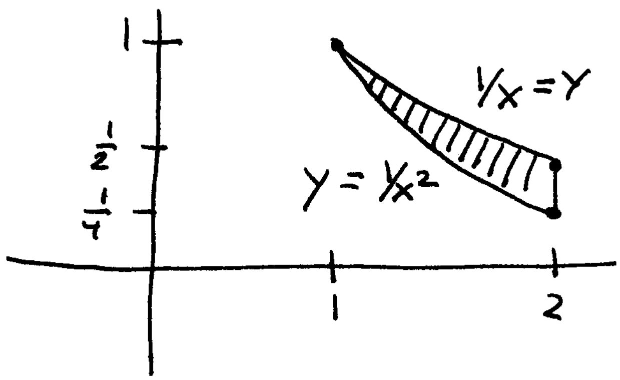
$$\frac{1}{2}\int (u-1)u^{1/2}\,du = \frac{1}{2}\int u^{3/2} - u^{1/2}\,du = \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$$

c) $\int \frac{3}{9+4x^2}\,dx = \frac{1}{2}\arctan\left(\frac{2x}{3}\right) + C$

$$\int \frac{3}{9+4x^2}\,dx = \frac{1}{3}\int \frac{dx}{1+\frac{4}{9}x^2}. \quad \text{Let } u = \frac{2x}{3},\ du = \frac{2}{3}dx$$

$$= \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan(u) + C = \frac{1}{2}\arctan\left(\frac{2x}{3}\right) + C$$

4[14P]) a) Sketch the region enclosed by the curves $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $x = 2$.

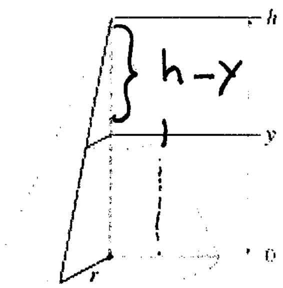


$\frac{1}{x} = \frac{1}{x^2}$ for $x = 1$

b) Find the area of the region in part (a). Area = $\ln(2) - \frac{1}{2}$

$$\int_1^2 \frac{1}{x} - \frac{1}{x^2}\,dx = \ln x + \frac{1}{x}\Big]_1^2 = \ln(2) + \frac{1}{2} - 1 = \ln(2) - \frac{1}{2}$$

5[14P]) Let V be the volume of a right circular cone of height $h = 4$ whose base is a circle of radius $r = 2$.



a) Find the area $A(y)$ of the horizontal cross section at a height y. $A(y) =$ $\pi\left(2 - \frac{y}{2}\right)^2 = \frac{\pi}{4}(4-y)^2$

$\frac{h}{r} = 2 = 4 - y/x$, $2x = 4 - y$: $x = 2 - y/2$

radius $= x$

b) Calculate V by integrating the cross-sectional areas. $V =$ 8π .

$$\frac{\pi}{4}\int_0^4 (4-y)^2\,dy = -\frac{\pi}{4}\int_4^0 u^2\,du$$

$u = 4 - y$

$du = -dy$

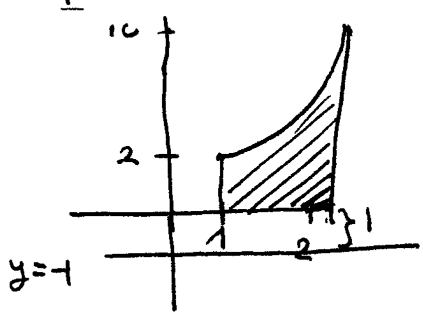
$$= \frac{\pi}{4}\int_0^4 u^2\,du = \frac{\pi}{8}u^3\Big]_0^4$$

$$= 8\pi$$

In the following three problems set up, but do NOT evaluate, an integral needed to find the volume. Do not forget the limits of integration:

6[10P]) Set up an integral for the volume of the solid obtained by rotating the region under the graph of the function $f(x) = 3x^2 - x$ over the interval $[1, 2]$ about the axis $y = -1$.

$V = \int_1^2 \pi \left((3x^2 - x + 1)^2 - 1 \right) dx$ (you can also simplify this)



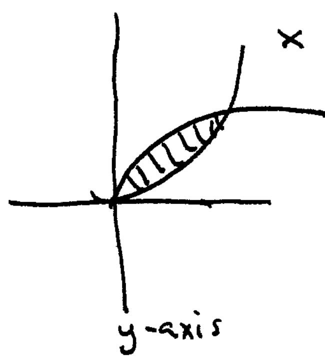
$R = f(x) + 1 = 3x^2 - x + 1$

$r = 1$

7[10P]) Set up an integral for the the volume of the solid obtained by rotating the region enclosed by the graphs $x = \sqrt{y}$ and $x = y^2$ about the y-axis.

$V = \pi \int_0^1 y - y^2 \, dy$

$\pi \int_0^1 y - y^4 \, dy$



$x = \sqrt{y}$

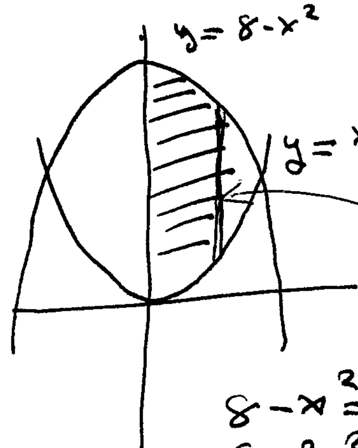
$x = y^2$

$\sqrt{y} = y^2$, $\underline{y = 1}$

8[10P]) Set up an integral needed to compute the volume of the solid obtained by rotating the region enclosed by the graphs of the functions $y = x^2$, $y = 8 - x^2$ and $x = 0$ about the y-axis by using the Shell Method.

$V = \int_0^2 2\pi x \left(8 - 2x^2\right) dx$

$= 2\pi \int_0^2 x(8 - 2x^2) dx$



$8 - x^2 - x^2$

$= 8 - 2x^2$

$8 - x^2 = x^2$

$8 = 2x^2$, $x = 2$