Problems for the wavelet class

You have to turn in at **least** the solution to **one** problem every week. The problems are due in the beginning of class each Friday, i.e., **the deadline is Friday at 10:40**. You can turn in more than one problem from each excersise set, but I will only grade one problem in all details.

Exercise set 1. Due Friday Feb. 7

1) Evaluate the Fourier transform of the 1-periodic function given by

$$f(t) = t\,, \quad -1/2 \leq t < 1/2\,.$$

Simplify the Fourier series by groupping together the even and odd n's and write the outcome as a series involing cos and sin.

2) Evaluate the Fourier transform of the 1-periodic function

$$f(t) = t\,. \quad 0 \leq t < 1\,.$$

Simplify the Fourier series by groupping together the even and odd n's and write the outcome as a series involing cos and sin.

Remark 1 If you have a graphical software so that you can plot the partial Fourier series $\sum_{n=-N}^{N} \hat{f}(n) e^{2\pi i n t}$ for different N's do that and discuss the result.

Remark 2 Notice that the functions in exercise 1 and exercise 2 agree for $0 \leq t < 1/2$ but the Fourier coefficients are not the same. This is therefore one example on how **global** behaviour of the functions affects the Fourier transform.

Exercise set 2. Due Friday Feb. 14

1) Let $f \in L^2(\mathbb{R})$ and $g \in C_c^\infty(\mathbb{R})$, where C_c^∞ stands for *smooth* and with *compact support*. Show that $f * g$ is smooth and that for all $n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ we have

$$(f * g)^{(n)} = f * g^{(n)} .$$

2) Show the following: Let $f \in L^1(\mathbb{R}^n)$ and $g \in C_c^\infty(\mathbb{R}^n)$. Assume that $g(x) \geq 0$, $g(0) \neq 0$ and $\int g(x)\, dx = 1$. For $t > 0$ define

$$g_t(x) := t^{-n} g(x/t) .$$

Show that $\int g_t(x) dx = 1$ and that in the L^1-norm.

$$\lim_{t \to 0} f * g_t = f .$$

3) Let $A < B$ be arbitrary. Evaluate the Fourier transform of $\chi_{[A,B]}$.