MATH 2025, fall 2009 Quiz 3 Name____________________

Please show all work, the correct arguments counts for half of the points! *(Please print)*

1[10P]) Which of the following sets of vectors is linearly independent and which are not?

a) $V = \mathbb{R}^2$, $(1,1)$, $(2,1)$.

b) $V = \mathbb{R}^3$, $(1,1,0)$, $(1,-1,1)$, $(2,1,1)$, $(-1,2,4)$.

c) $V = \mathbb{R}^4$, $(1,0,1,-1)$, $(2,0,-1,1)$, $(0,1,1,1)$.

d) $V = C([0,1])$ x, x^2. $1+x^2$.

2[5P]) Write the vector $(3,3,6)$ as a linear combination of $(1,-1,0)$, $(1,1,1)$, and $(1,1,-2)$.

3[5P] Which of the following sets of vectors is a basis for the given vector space?

a) $V = \mathbb{R}^2$, $(1,4)$, $(-2,-8)$.

b) $V = \mathbb{R}^3$, $(1,1,-2)$, $(1,1,1)$, $(1,-1,0)$.

c) $V = \mathbb{R}^3$, $(1,3,1)$, $(1,-1,2)$.

d) $V = \mathbb{R}^3$. $(1,2,1)$, $(2,-1,0)$, $(2,-1,-1)$, $(1,4,-1)$.

Solutions to quiz #3

1) a) Linearly independent.

One way: Not on a line.

Another way: If $c_1(1,1)+c_2(2,1)=(0,0)$ then

$$\left.\begin{array}{l} c_1+2c_2=0 \\ c_1+c_2=0 \end{array}\right\} \text{ substract: } \quad c_2=0$$

insert $c_2=0$ in one of the equation to get $c_1=0$.

Third way: $\det\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = 1-2=-1\neq 0.$

b) 4 vectors in $\mathbb{R}^4$ are always linearly dependent.

c) Linearly independent: They are orthogonal to each other!

d) Linearly independent: $c_1 x + c_2 x^2 + c_3(1+x^2)=0$ means: what ever number $x\in[0,1]$ I insert the outcome is always 0. Take $x=0$, then

$c_3=0$, so $c_1 x + c_2 x^2 = x(c_1+c_2 x)=0$

Differentiating the zero function is zero, so

$$c_1+2c_2 x=0$$

Take $x=0$, then $c_1=0$. Finally, by differentiating one more time gives $c_2=0$

2) The vectors are orthogonal. Thus

$$(3,3,6) = \frac{\overbrace{(3,3,6)\cdot(1,-1,0)}^{=0}}{\|(1,-1,0)\|^2}(1,-1,0) + \frac{(3,3,6)\cdot(1,1,1)}{\|(1,1,1)\|^2}(1,1,1) + \frac{(3,3,6)\cdot(1,1,-2)}{\|(1,1,-2)\|^2}(1,1,-2)$$

$$= \frac{12}{3}(1,1,1) + \frac{-6}{6}(1,1,-2)$$

$$= \underline{\underline{4(1,1,1) - (1,1,-2)}}.$$

3) a) $(-2,-8) = (-2)(1,2)$ linearly dependent. <u>Not a basis</u>

b) pairwise orthogonal, therefore linearly independent. Any set of 3 linearly independent vectors in $\mathbb{R}^3$ is a basis. (In this case we see that

$$(x,y,z) = \frac{x+y-2z}{6}(1,1,-2) + \frac{x+y+z}{3}(1,1,1) + \frac{x-y}{2}(1,-1,0)$$

so it is a spanning set.)

c) <u>Not a basis</u>, we need 3 vectors.

d) <u>Not a basis</u>. 4 vectors in $\mathbb{R}^3$ ar always linearly dependent.