Math 2057-5 Quiz #4 (Fall 2005) **Name:** ____________________

Evaluate the double integrals:

1) $\int_0^1 \int_0^{x^2} (x+2y)\,dydx = \underline{9/20}$

The first integral is:

$$\int_0^{x^2} (x+2y)\,dy = \left[xy+y^2\right]_0^{x^2} = x^3+x^4\,.$$

We therefore get:

$$\begin{aligned}\int_0^1 \int_0^{x^2} (x+2y)\,dydx &= \int_0^1 x^3+x^4\,dx \\ &= \frac{1}{4}x^4+\frac{1}{5}x^5|_0^1 \\ &= \frac{1}{4}+\frac{1}{5} \\ &= \frac{5+4}{20} = \frac{9}{20}\,.\end{aligned}$$

2) $\iint_D e^{y^2}\,dA = \underline{\frac{1}{2}(e-1)}$, where $D = \{(x,y) \mid 0 \le y \le 1 \;\; 0 \le x \le y\}$.

The integral can be written as $\int_0^1 \int_0^y e^{y^2}\,dxdy$. The first integral is then

$$\int_0^y e^{y^2}\,dy = xe^{y^2}|_0^y = ye^{y^2}\,.$$

The second integral is

$$\begin{aligned}\int_0^1 ye^{y^2}\,dy &= \frac{1}{2}\int_0^1 e^u\,du \quad (u=y^2\,,\; du=2ydy) \\ &= \frac{1}{2}e^u|_0^1 \\ &= \frac{1}{2}(e-1)\,.\end{aligned}$$

3) $\iint_D e^{-x^2-y^2}\,dA = \underline{\pi(1-e^{-4})}$, where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis.

Using polar-coordinates this integral becomes

$$\int_{-\pi/2}^{\pi/2}\int_0^2 e^{-r^2} rdrd\theta = \int_0^2 \int_{-\pi/2} \pi/2 e^{-r^2} rd\theta dr = \pi\int_0^2 e^{-r^2} rdr\,.$$

Making the change of variable $u=-r^2$, $du=-2rdr$, the last integral becomes

$$-\int_0^{-4} e^u\,du = \int_{-4}^0 e^u\,du = 1-e^{-4}\,.$$