Math 2057-5 Quiz #5 (Fall 2005) **Name:** ______________________

Evaluate the following triple integrals:

1) $\iiint_E xy\,dV =$ <u>65/168</u> where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

<u>Solution</u>: The integral is

$$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} xy \, dz\,dy\,dx$$

1th integral $xy \int_0^{1+x+y} dz = xyz\Big]_0^{1+x+y} = xy + x^2y + xy^2$

2th integral: $\int_0^{\sqrt{x}} xy + x^2y + xy^2\,dy = \frac{1}{2}(x+x^2)y^2 + \frac{1}{3}xy^3\Big]_0^{\sqrt{x}} = \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{3}x^{5/2}$

last integral $\int_0^1 \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{x^{5/2}}{3}\,dx = \frac{1}{6}x^3 + \frac{1}{8}x^4 + \frac{2}{21}x^{7/2}\Big]_0^1 = \frac{1}{6} + \frac{1}{8} + \frac{2}{21} = \frac{65}{168}$

2) $\iiint_E x\,dV =$ <u>1/4</u>, where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

The integral is

$$\int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} x\,dz\,dy\,dx$$

1th integral $\int_0^{3-3x-\frac{3}{2}y} x\,dz = 3x - 3x^2 - \frac{3}{2}xy$

2th integral $\int_0^{2-2x} 3x - 3x^2 - \frac{3}{2}xy\,dy = 3xy - 3x^2y - \frac{3}{4}xy^2\Big]_0^{2(1-x)}$

$= 3x - 6x^2 + 3x^3$

3th integral: $\int_0^1 3x - 6x^2 + 3x^3\,dx = \frac{3}{2}x^2 - 2x^3 + \frac{3}{4}x^4\Big]_0^1$

$= \frac{3}{2} - 2 + \frac{3}{4} = \frac{6-8+3}{4} = \underline{\underline{\frac{1}{4}}}$