**Math 2057-5 Quiz #6 (Fall 2005)** **Name:** ____________________

**1[3P])** Find the gradient vector field corresponding to the potential function $\varphi(\mathbf{x}) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$.

**Solution:** The gradient is

$$\nabla\varphi(\mathbf{x}) = < \frac{\partial\varphi}{\partial x}(\mathbf{x}), \frac{\partial\varphi}{\partial y}(\mathbf{x}), \frac{\partial\varphi}{\partial z}(\mathbf{x}) > \ .$$

We have

$$\begin{aligned} \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) &= \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2} \\ &= \frac{-1}{2}\cdot 2x\cdot(x^2+y^2+z^2)^{-3/2} \\ &= -\frac{x}{(\sqrt{x^2+y^2+z^2})^3}\,. \end{aligned}$$

Then doing the same calculation for the other variables shows that

$$\nabla\varphi(\mathbf{x}) = \left\langle -\frac{x}{(x^2+y^2+z^2)^{3/2}}, -\frac{y}{(x^2+y^2+z^2)^{3/2}}, -\frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle = -\frac{\mathbf{x}}{|\mathbf{x}|^3}$$

**2[3.5P])** Evaluate the line integral $\int_C xy\,ds$ where $C$ is the arc of the parabola $y = x^2$ from $(-1,1)$ to $(1,1)$.

**Solution:** We use $x$ as parameter, so the curve is described by $C$: $x = x$, $y = x^2$, $-1 \le x \le 1$, and $ds = \sqrt{1+4x^2}\ dx$. The integral is therefore

$$\int_C xy\,ds = \int_{-1}^{1} x^3\sqrt{1+4x^2}\,dx\,.$$

The function $x \mapsto x^3\sqrt{1+4x^2}$ is odd and we integrate from $-1$ to $1$. Hence the integral is zero. The final answer is therefor

$$\underline{\int_C xy\,ds = 0\,.}$$

**3[3.5P])** Evaluate the line integral $\int_C xy\,dx + (x-y)\,dy$ where $C$ consists of the line segment from $(0,0)$ to $(1,2)$.

**Solution:** The line segment can be parametrized as

$$x = t,\ y = 2t,\ 0 \le t \le 1\,.$$

Then $dx = dt$ and $dy = 2dt$. The integral is therefore

$$\begin{aligned}
\int_C xy\,dx + (x-y)\,dy &= \int_0^1 (t \cdot 2t + 2(t-2t))\,dt \\
&= \int_0^1 2t^2 - 2t\,dt \\
&= \left[\frac{2t^3}{3} - t^2\right]_0^1 \\
&= \frac{2}{3} - 1 \\
&= -1/3\,.
\end{aligned}$$

The final answer is therefore

$$\underline{\int_C xy\,dx + (x-y)\,dy = -1/3\,.}$$