Math 7390, Applied harmonic analysis and wavelets

Textbook: C. Blatter: Wavelets: A Primer. A.K. Peters.
Time and location: 10:40–11:30, M-W-Fr in Lockett 237
Instructor: Gestur Ólafsson, Lockett 322.

Harmonic analysis deals with the problem of decomposing functions into simpler functions. What "simpler functions" means depends on the situation, and which properties we are looking for. The classical situation is the Fourier analysis which decomposes arbitrary functions simultaneously into waves and into eigenfunctions of differential operators with constant coefficients. The object of Fourier series is to expand arbitrary periodic $\mathbf{L}^2$-function in terms of the orthogonal basis $(2\pi)^{-1/2}e^{ikx}$, $k \in \mathbb{Z}$. Those functions have exact localization in the frequency, i.e., the Fourier variable, but have no precise localization in space. This makes Fourier series not always the right tool for signal analysis, where both good localization in the frequency and space parameter is reqiuered. In the last 20 years a new theory has emerged that deals exactly with this problem. The wavelet theory tries to construct an orthonormal basis that has localization properties both in space and frequency. It uses both translation and dilation to zoom into given part of the spatial variable. Wavelet theory lies on the boundary between

1. Mathematics, in particular harmonic analysis
2. Signal processing,
3. Image processing
4. Scientific calculation

In this corse we will give an overview of the wavelet theory and its applications. This course will deal with the following topics,

1. Introduction to Fourier series and integrals.
2. Hilbert spaces, orthonormal basis and Riesz basis. Continuous operators between Hilbert spaces.
3. Multiresolution approximations of $\mathbf{L}^2(\mathbb{R})$.
4. Haar wavelets and Daubechies Wavelets.
5. Construction of wavelets in one dimension.
6. Signal compression and denoising.

This is not a class on integration theory and Hilbert spaces, so we expect the student to have some minimal knowledge here. But this course is also for students in applied science, so that we will give a short overview, and then get to the central topic of this class, wavelets and their application. The student should know 7312 or equivalent.