**Math 2025, Test #2 . Fall 2002 Name:** ______________________________

**1[8P])** Apply the two dimensional Haar wavelet transform to the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$.

**2[12P])** Appy the two dimensional Haar wavelet transform to the matrix $\begin{pmatrix} 4 & -2 & 11 & -1 \\ 2 & 0 & 5 & -3 \\ 20 & -4 & 2 & -2 \\ 8 & 2 & -4 & -4 \end{pmatrix}$

**3[8P])** Let $z = 2 + 3i$ and $w = \frac{1}{2+i}$. Evaluate the following:

**a)** $z \cdot w = \frac{7}{5} + \frac{4}{5}i$

**b)** $\bar{z} =$

**c)** $z^2 =$

**d)** $|w|^2 =$

**4[8P])** Evaluate the following multiplication of matrices:

**a)** $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 1 & -5 & 3 \end{bmatrix} =$

**b)** $\begin{bmatrix} 2 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ -1 & 2 \\ 4 & 3 \end{bmatrix} =$

**5[28P])** Determine if the each of the following sets is a vector space or not, and **state why:**

a) The space of polynomials of degree $\leq 5$, i.e., $V = \left\{\sum_{j=0}^{5} a_j x^j \mid \forall j \ : \ a_j \in \mathbb{R}\right\}$;

**b)** $V = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + x^2 - y + 2z = 0\}$;

**c)** $V = \left\{f \in C([-1, 1]) \mid \int_{-1}^{1} f(t)\, dt = 0\right\}$;

**d)** The space $V_3$ of all functions on the interval $[0, 1)$ of the form $\sum_{j=0}^{7} a_j \psi_j^3$, with arbitrary real numbers $a_1, \ldots, a_7$. Here $\psi_j^3(t) = \psi(8t - j)$.

**e)** Let $A$ be a $n \times m$ matrices and $V = \{\mathbf{x} = [x_1, \ldots, x_n] \in \mathbb{R}^n \mid \mathbf{x}A = \mathbf{0}\}$;

**f)** $V = \{u \in U \mid T(u) = v\}$ where $U$ and $W$ are vector spaces, $T : U \to W$ is linear and $y \in W$, $y \neq 0$.

**g)** The space of functions on the real line $\mathbb{R}$ that are solutions to the differential equation $y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_0 y = 0$, i.e.,$V = \{y \in C^\infty(\mathbb{R}) \mid y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_0 y = 0\}$.

**6[24P])** Determine if the following maps are linear or not, **state why:**

**a)** $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (2x + y - z, xy)$.

**b)** $V$ the space of polynomials of degree $\leq 5$ and $W$ the space of polynomials of degree $\leq 4$, $T(p)(x) = 2p'(x) + 3p''(x)$ .

**c)** $T : \mathbb{R}^4 \to \mathbb{R}$; $T(x_1, x_2, x_3, x_4) = 2x_1 + x_2 - 3x_3 + 4x_4$.

**d)** Let $V_N = \left\{\sum_{j=0}^{2^N-1} s_j \varphi_j^N \mid \forall j = 0, \ldots, 2^N - 1 : a_j \in \mathbb{R}\right\}$ and $T : V_N \to V_{N-1}$ given by

$$T\left(\sum_{j=0}^{2^N-1} s_j \varphi_j^N\right) = \sum_{j=0}^{2^{N-1}-1} \frac{s_{2j} + s_{2j+1}}{2} \varphi_j^{N-1} .$$

**e)** $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (2x + y - 3z, 3x + y + 2, x - 4y + z)$.

**f)** $T : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$, $T(f) = f'' + f' \cdot f$.

**7[12P])** In the following problems, evaluate the given linear map $T$ at the given point:

**a)** $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (2x + 3y, -x + 4y)$, $(x, y, z) = (2, -1, 4)$;

**b)** $T : \mathbb{C}^3 \to \mathbb{C}^3$, $T(z_1, z_2, z_3) = ((1 + i)z_1 + 2z_2 - iz_3, z_1 + (1 - i)z_2, z_2 - \frac{1}{1+i}z_3)$, $(z_1, z_2, z_3) = (i, 1 + i, 2 + i)$.

**c)** $T : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$, $T(f) = f'' + 4f$, $f = 2\cos(x) + \sin(x) + e^x$.

**d)** $T : C([-1, 1]) \to \mathbb{R}$, $T(f) = \int_{-1}^{1} f(t)\ dt$, $f(t) = t^2 + t + \cos(\pi t)$.