MATH 1552-3, Summer 2011 Name: Solutions

Test 3, Wednesday, July 20, 2011. For partial credit, show all your work!

1[24P]) Let $c(t) = (t^2+1, t^2-2t)$.

a) Find the x and y coordinates at time $t=2$. $x =$ 5, $y =$ 0

b) Evaluate $\frac{dy}{dx}$ at the point $t=2$. $\frac{dy}{dx} =$ $1/2$.

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2t-2}{2t}, \quad t=2: \quad \frac{2}{4} = \frac{1}{2}$$

c) The equation of the tangent line through the point $c(2)$ is 1 $y =$ $\frac{1}{2}$ $x +$ $-5/2$

$$y - y_0 = m(x - x_0): \quad y = \frac{1}{2}(x-5) = \frac{1}{2}x - 5/2$$

2[7P]) The curve $r = \dfrac{10}{2\cos(\theta) - \sin(\theta)}$ represents a line. The equation of the line in form of x and y is: 1 $y =$ 2 $x +$ -10

multiply: $2r\cos\theta - r\sin\theta = 10, \quad 2x - y = 10$

3[27P]) A circle C has center at the origin and radius 2. The circle K has radius 2 and center at the point $(0,2)$.

a) Write the equation of both circles in polar coordinates.
C has the equation $r =$ 2 and K has the equation $r =$ $4\sin(\theta)$

b) Find the x, y coordinates of the points where the two circles intersects. $x =$ $\pm\sqrt{3}$, $y =$ 1

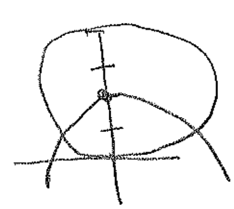
$2 = 4\sin(\theta), \quad \sin(\theta) = 1/2, \quad \cos(\theta) = \sqrt{3}/2, \quad x = 2\cos\theta$

$y = 2\sin(\theta)$

c) Set up the integral representing the area **outside** the circle C and **inside** the circle K.

$$\theta = \sin^{-1}(1/2) = \pi/6$$

$$\text{area} = \int_{\pi/6}^{\pi/2} 16\sin^2(\theta) - 4 \, d\theta$$



4)[**9P**]) Set up the integral for the length of the curve $r = 1 + \cos(\theta)$, $0 \le \theta \le 2\pi$.

$$\sqrt{2}\int_0^{2\pi} \sqrt{1+\cos\theta}\, d\theta$$

$$\text{Length} = \int_a^b \sqrt{(r')^2 + r^2}\, d\theta, \quad r' = -\sin(\theta), \quad r'^2 + r^2 = 1 + 2\cos\theta + \cos^2\theta + \sin^2\theta$$

5[**10P**]) The equation of the ellipse that has a center at $(6,3)$, a focus at $(2,3)$, and a vertex at $(1,3)$ is:

$$\frac{(x-6)^2}{25} + \frac{(y-3)^2}{9} = 1$$

$c = 6 - 2 = 4$

$a = 6 - 1 = 5$

$b^2 = a^2 - c^2 = 25 - 16 = 9$

6[**18P**]) Suppose that $P = (2,1,1)$ and $Q = (-1,3,1)$.

a) Find $\overrightarrow{PQ} =$ $\langle -3, 2, 0 \rangle$ and $\|\overrightarrow{PQ}\| =$ $\sqrt{13}$

$9 + 4 = 13$

b) The equation of the line through the points P and Q is $\langle 2,1,1 \rangle + t\langle -3,2,0 \rangle$ or

$x = 2 - 3t$

$y = 1 + 2t$

$z = 1$

7[**18P**]) Given two lines with parameters $\mathbf{r}_1(t) = \langle 10, 6, 5 \rangle + t\langle -2, -2, 1 \rangle$ and $\mathbf{r}_2(t) = \langle 3, 5, 1 \rangle + t\langle 1, -1, 2 \rangle$.

a) Find the point of intersection, P, of the lines $\mathbf{r}_1$ and $\mathbf{r}_2$. $P =$ $(6, 2, 7)$

$10 - 2t = 3 + s$
$6 - 2t = 5 - s$
$5 + t = 1 + 2s$

$s = 7 - 2t$
$= -1 + 2t$

$0 = -8 + 4t$
$t = 2$
$s = 7 - 4 = 3$

Test last equation
$5 + 2 = 7$
$1 + 6 = 7$
Insert: $P = (10-4, 6-4, 5+2)$

b) Write the equation of the line through the point $Q = (1,1,1)$ and parallel to the line given by $\mathbf{r}_1(t)$.

$\vec{r}_3(t) = \langle 1,1,1 \rangle + t\langle -2,-2,1 \rangle$

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a) Note $\vec{r}_1$ was changed to $r_1(t) = \langle 6, 6, 2 \rangle + t\langle -2,-2,1 \rangle$. Now the solution is $t = s = 1$ so $P = (4, 4, 3)$

8[18P]) Let $\mathbf{u} = \langle 2, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, -1, 1 \rangle$.

a) The unit vector $\mathbf{e_u}$ in the direction of $\mathbf{u}$ is $\mathbf{e_u} = \frac{1}{\sqrt{6}} \langle 2, 1, 1 \rangle$

$$e_{\vec{u}} = \frac{\vec{u}}{\|\vec{u}\|}, \quad \|\vec{u}\| = \sqrt{4+1+1} = \sqrt{6}$$

b) Find the projection $\text{proj}_{\mathbf{u}}(\mathbf{v}) = \langle \frac{2}{3}, \frac{1}{3}, \frac{1}{3} \rangle$

$$\text{proj}_{\vec{u}}(\vec{v}) = (\vec{v} \cdot \vec{e_{\vec{u}}})\, e_{\vec{u}} = \frac{1}{6}(2-1+1) \cdot \langle 2, 1, 1 \rangle$$

9[9P]) Write the equation of the line through the point $P = (1, -1, 2)$ with normal $\mathbf{n} = (1, -2, 1)$.
The equation is $\underline{2}\, x + \underline{-4}\, y + \underline{2}\, z = 10$.

Equation: $\vec{n} \cdot (\vec{x} - \vec{x}_0) = 0$ or $\vec{n} \cdot \vec{x} = \vec{n} \cdot x_0 = 1+2+2=5$

$x - 2y + z = 1+2+2 = 5$

10[10P]) Find the equation of the plane through the points $P = (1, 0, 2)$, $Q = (1, 1, 2)$ and $R = (-1, 1, 0)$.

$\vec{n} = \vec{PQ} \times \vec{PR} = \langle 0, 1, 0 \rangle \times \langle -2, 1, -2 \rangle$ or any other vector orthogonal to $\langle 0, 1, 0 \rangle$, $\langle -2, 1, -2 \rangle$. One such vector is $\langle 1, 0, -1 \rangle$. So the equation is

$$x - z = -1 \quad \text{or} \quad -x + z = 1$$