Scattering of Electromagnetic Waves by Periodic Waveguides: Resonance and Optimization

VIGRE Crew on Material Science
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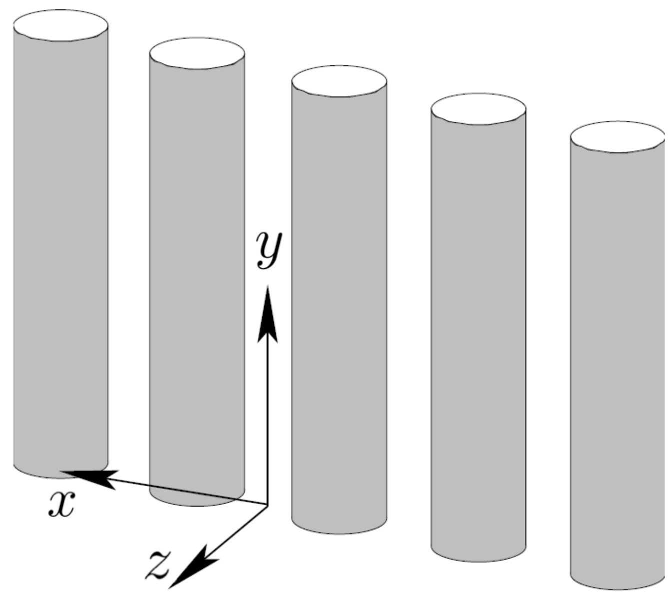
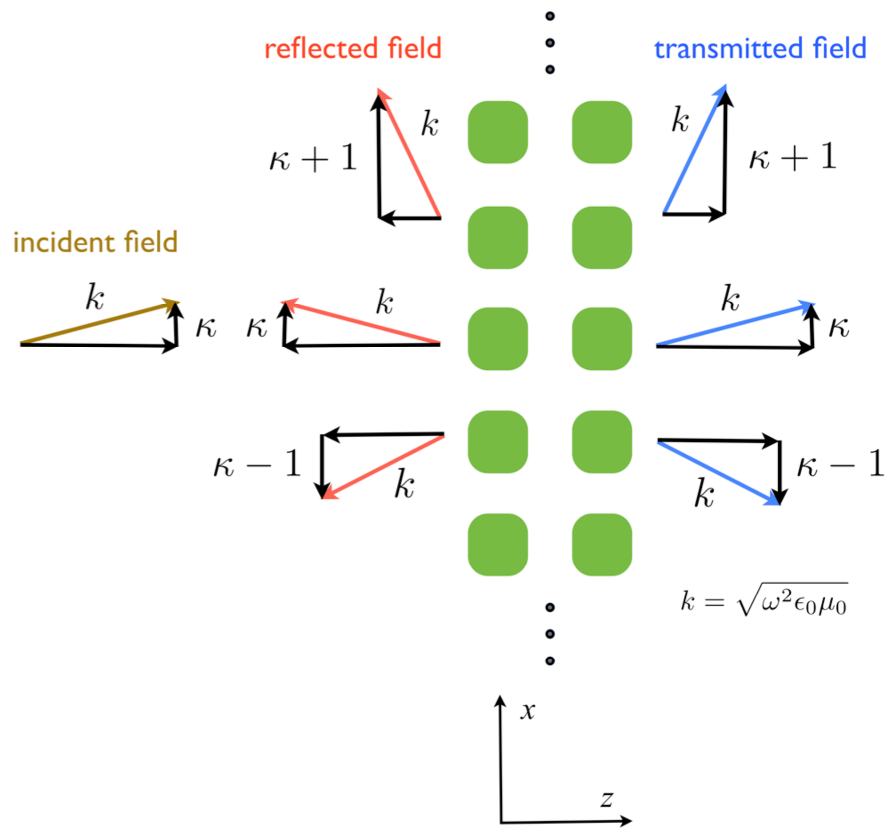
Scattering of electromagnetic waves by a periodic rod structure





An example of a periodic structure of dielectric rods.

The rods are infinitely tall in the z-direction, and the wall of rods is infinitely long in the x-direction.

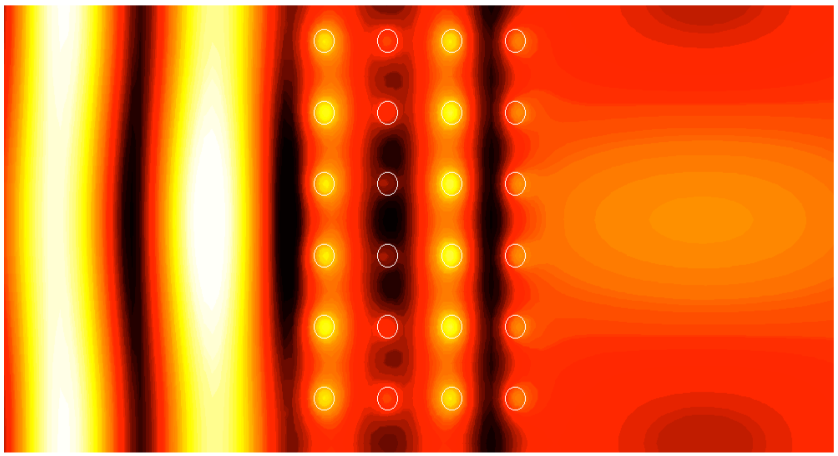
A cross-section of a periodic slab array of rods, two rods thick.

An incident plane wave strikes the slab from the left.

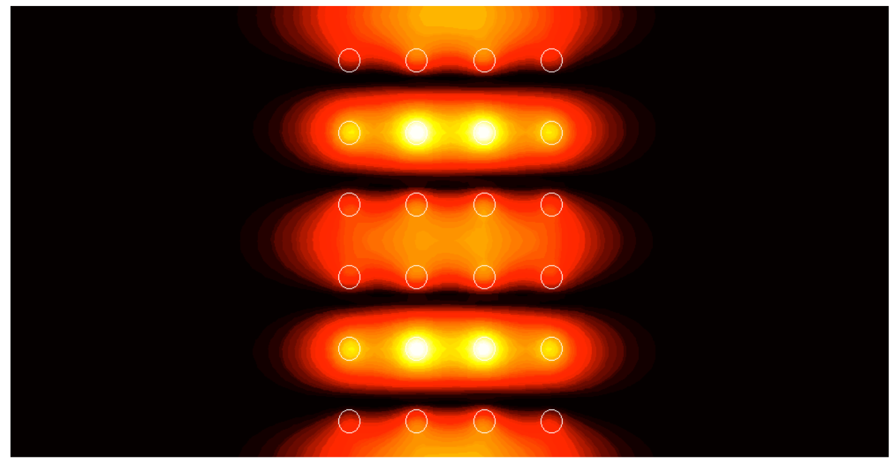
The field is reflected and transmitted in a finite number of directions. These are the *diffractive orders*.

Some simulations of scattering by a periodic array of rods

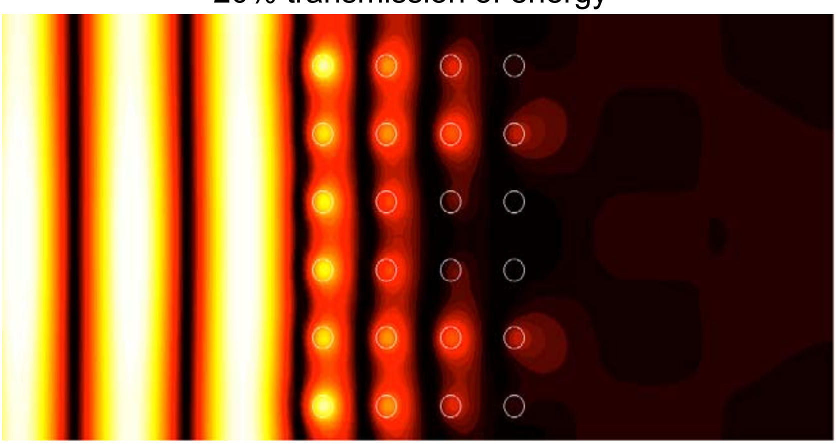
85% transmission and localization of the transmitted field



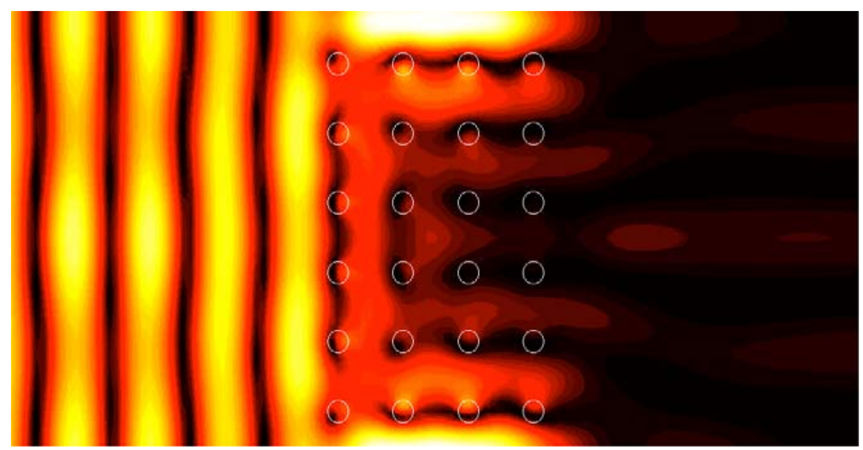
Resonance of an incident field with a guided mode.



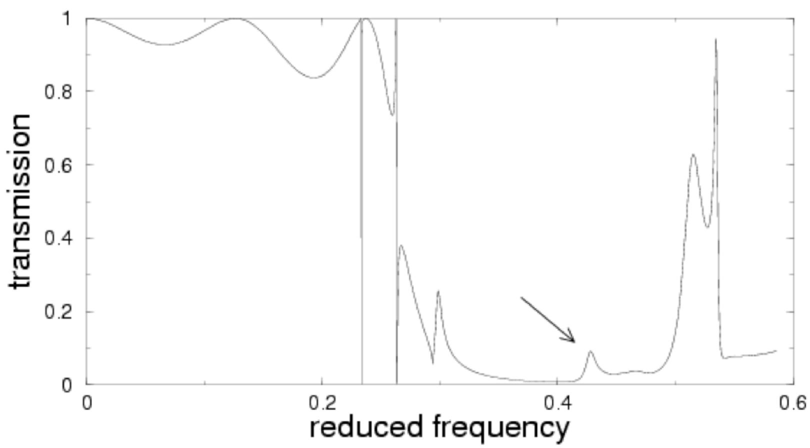
20% transmission of energy

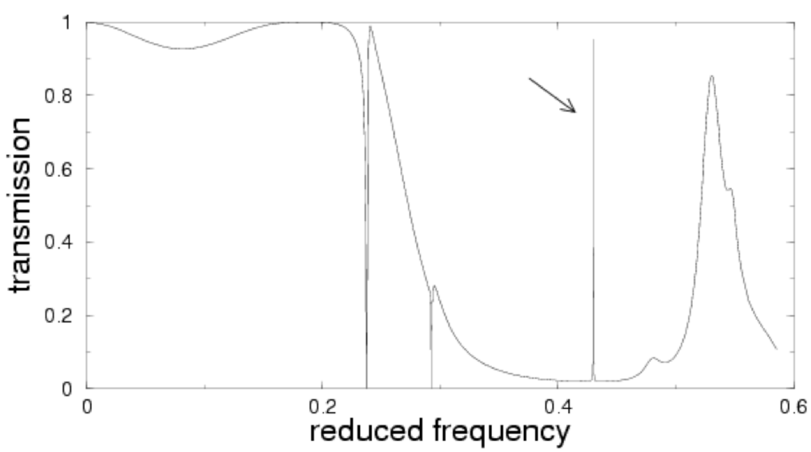


Resonance in a channel defect of the slab.

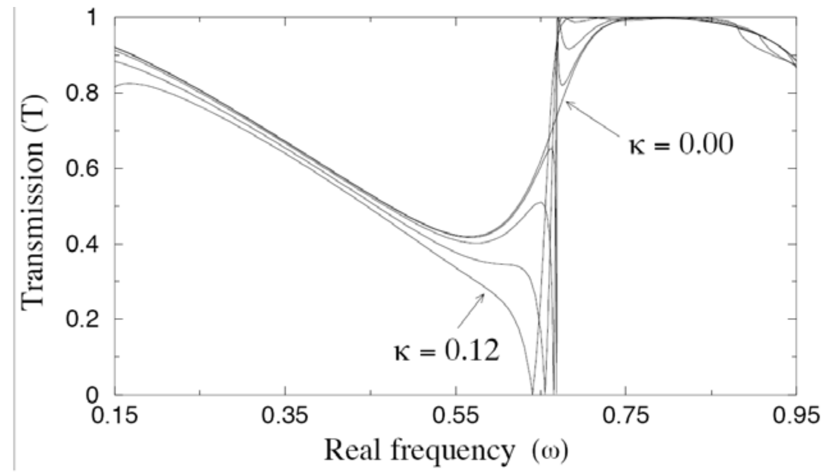


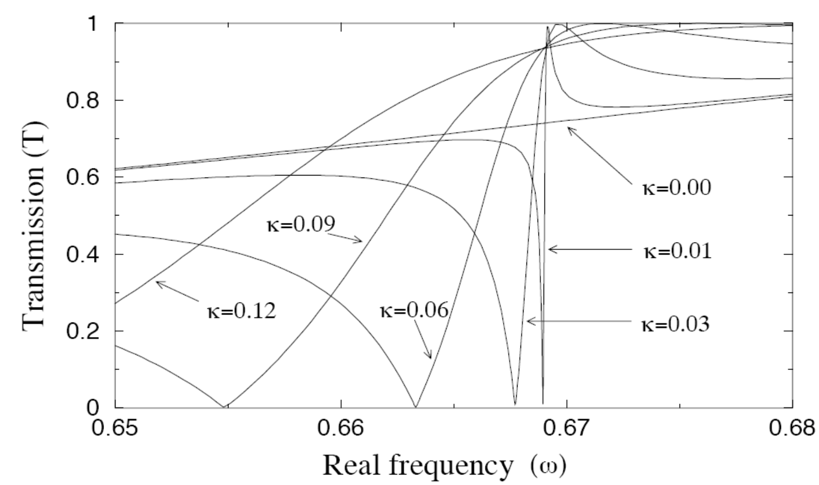
Optimization





Resonance





Mathematical Formulation

The governing equation for polarized EM or acoustic fields is the scalar **wave equation**

$$\epsilon U_{tt}(x, z; t) = \nabla \cdot \mu^{-1} \nabla U(x, z; t).$$

We are interested in **time-harmonic solutions**

$$U(x, z; t) = u(x, z) e^{i\omega t}.$$

If we insert this form into the wave equation, we obtain the **Helmholtz equation** for the spatial part of the field $u(x, z)$:

$$\nabla \cdot \mu^{-1} u(x, z) + \omega^2 \epsilon u(x, z) = 0.$$

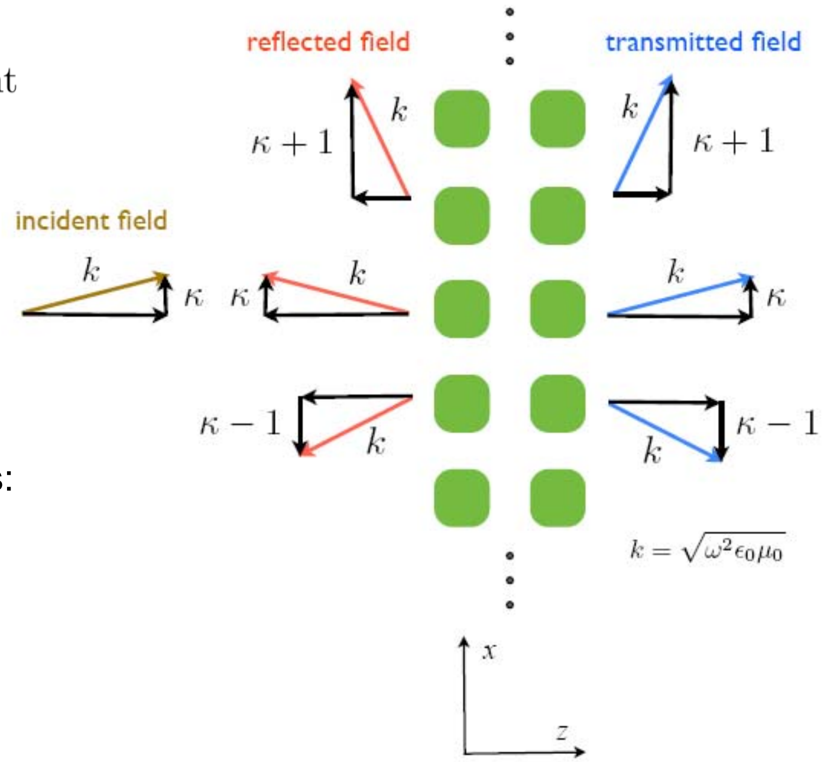
In addition, we consider fields that travel with a wavevector κ parallel to the slab and are modified periodically because of the structure. Such fields are called **pseudo-periodic** with Bloch wavenumber κ:

$$u(x, z) = \hat{u}(x, z) e^{i\kappa x} \quad \text{with } \hat{u}(x, z) \text{ periodic in } x.$$

Rayleigh-Bloch scattering by a periodic structure

Solution of the scattering problem:

$$u(x,z) = \begin{cases} e^{i(\kappa x + \eta_0 z)} + \sum_{m\in\mathbb{Z}} a_m e^{-i\eta_m z} e^{i(m+\kappa)x} & \text{to the left} \\ \sum_{m\in\mathbb{Z}} b_m e^{i\eta_m z} e^{i(m+\kappa)x} & \text{to the right} \end{cases}$$



The exponents in the *z*-direction must satisfy

$$\eta_m^2 + (m+\kappa)^2 = \epsilon_0\mu_0\omega^2$$

The spatial (Fourier) harmonics behave in three possible ways:

$$\begin{array}{ll} \epsilon_0\mu_0\omega^2 - |m+\kappa|^2 > 0 & \text{finitely many propagating } m \\ \epsilon_0\mu_0\omega^2 - |m+\kappa|^2 = 0 & \text{finitely many linear } m \\ \epsilon_0\mu_0\omega^2 - |m+\kappa|^2 < 0 & \text{infinitely many evanescent } m \end{array}$$

Guided modes:

A nonzero solution of the homogeneous problem (no incident field) has *only evanescent harmonics* And therefore is exponentially localized to the structure. A special relation between frequency and wavenumber (the dispersion relation) is necessary for a guided mode.