Resonant Scattering by an Open Waveguide: Dependence on Angle of Incidence and Geometry

Jennifer Ribbeck, Katherine Smith, Clayton Weeks

Introduction

Abstract: We investigate anomalies in the transmission of plane waves across a periodic waveguide produced by resonant interaction with the modes of the guide. Through numerical computations and direct calculations for a discrete model, we learn how perturbations of the angle of incidence and the structure itself affect these anomalies. The discrete model gives an exact formula that matches the numerical results for the continuous problem. This work builds on that of Ptitsyna, Shipman, and Venakides.

Resonance: tendency for a structure to amplify fields, associated with characteristic frequencies specific to the structure

Guided (Trapped) Mode: a pattern of wave motion that decays external to the structure and does not lose energy by radiation

Introduction (cont.)

Periodic Structure: structure with repeating identical units

Figure 1. Periodic Scatterer

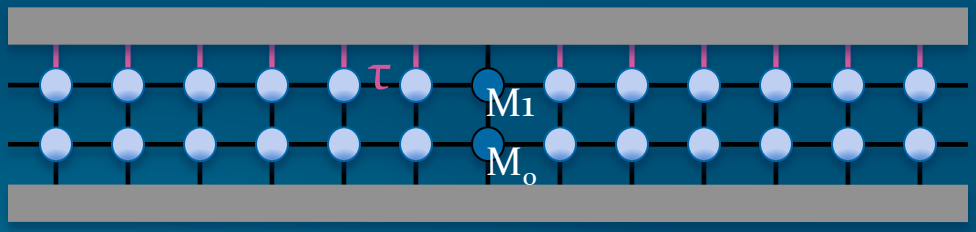
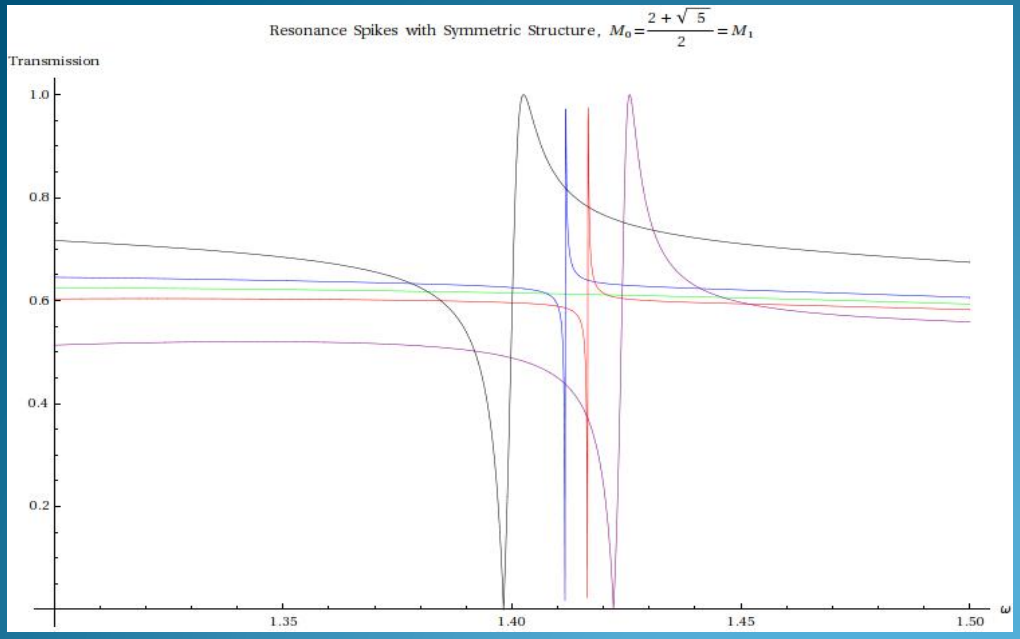


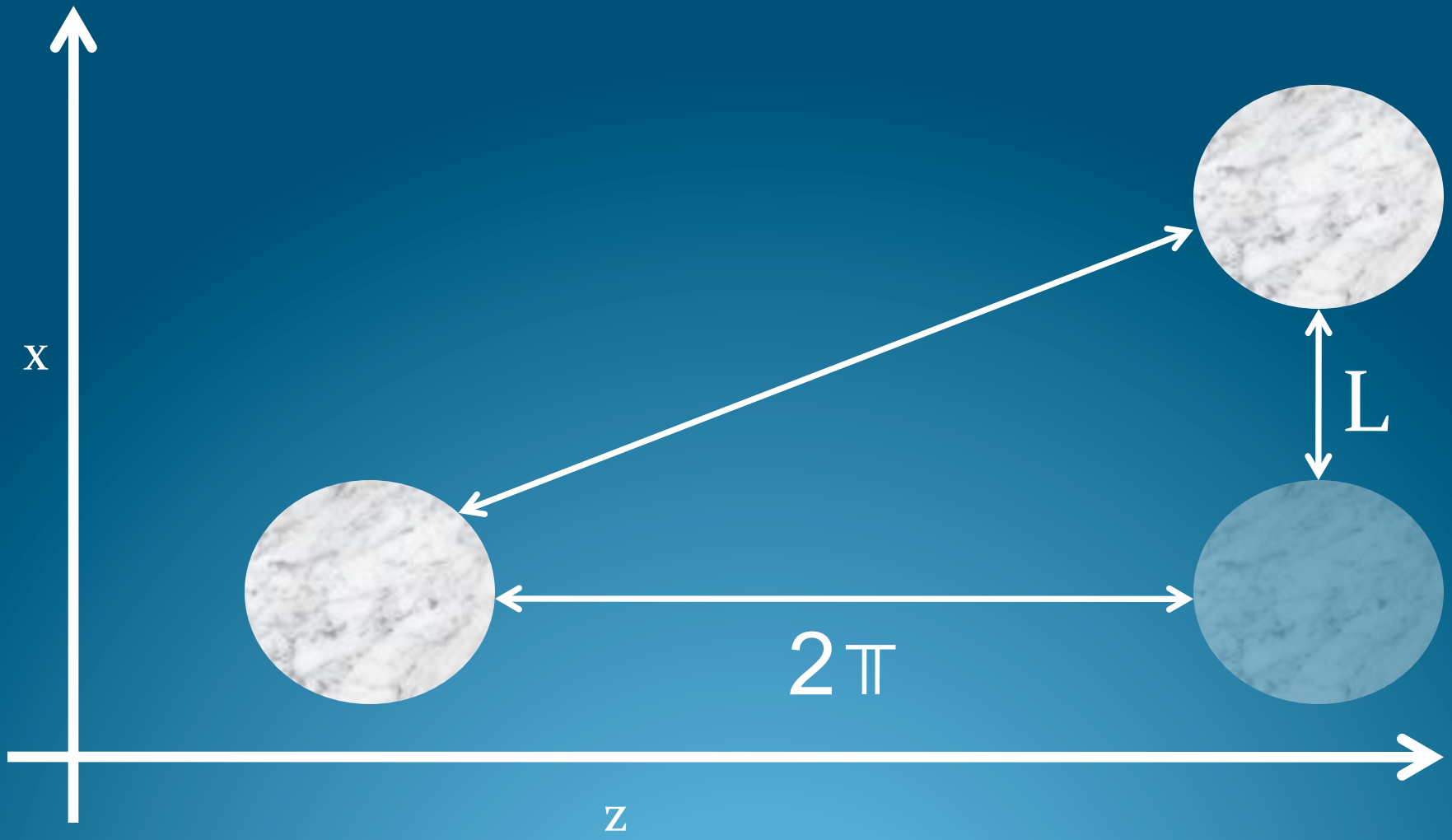
Figure 2. Discrete Model

Sample Transmission Graph:

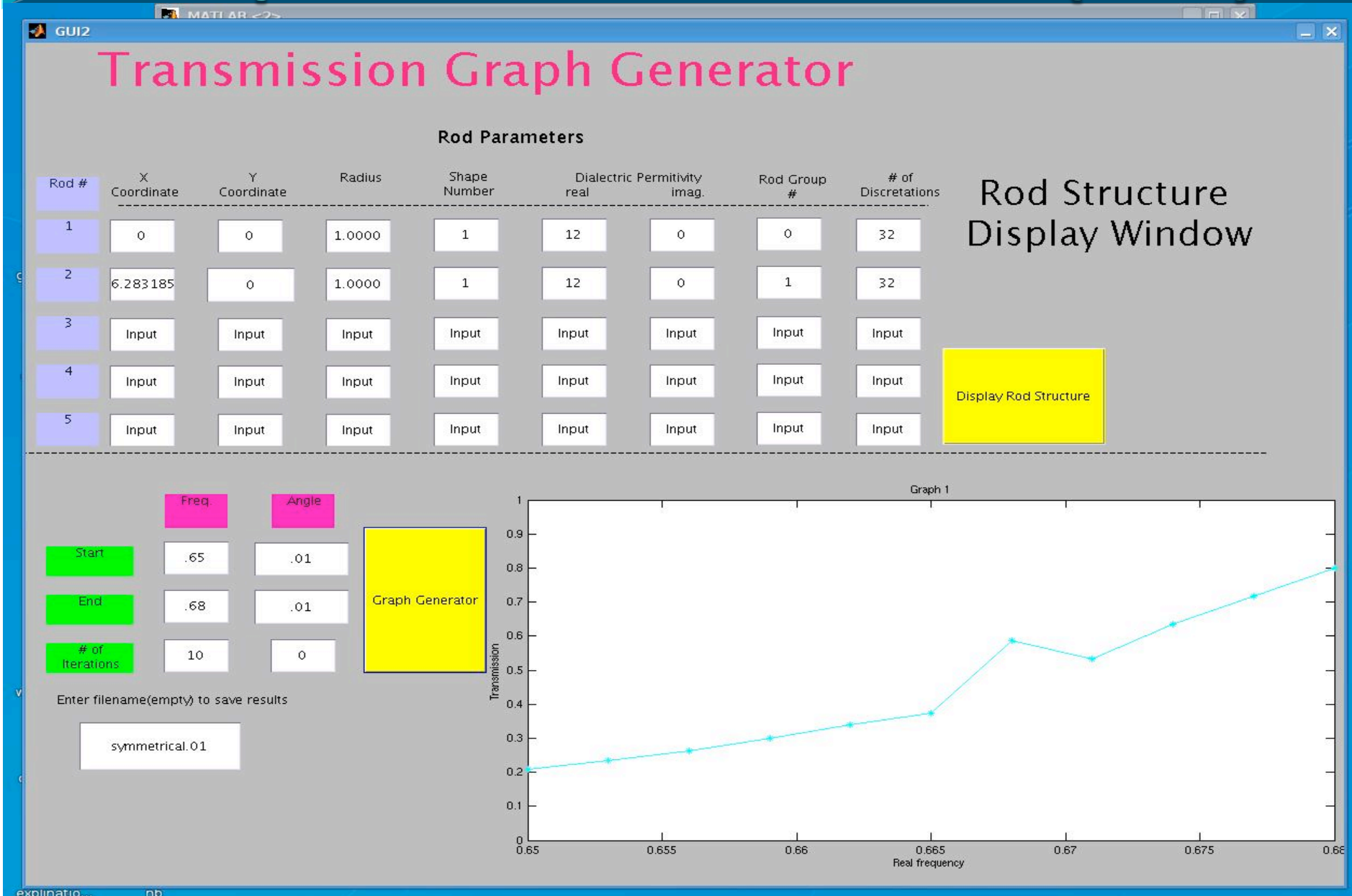


Spikes: dips and peaks in transmission that relate to the angle of incidence of an EM field, the perturbation of periodicity and the characteristic frequency of a periodic structure.

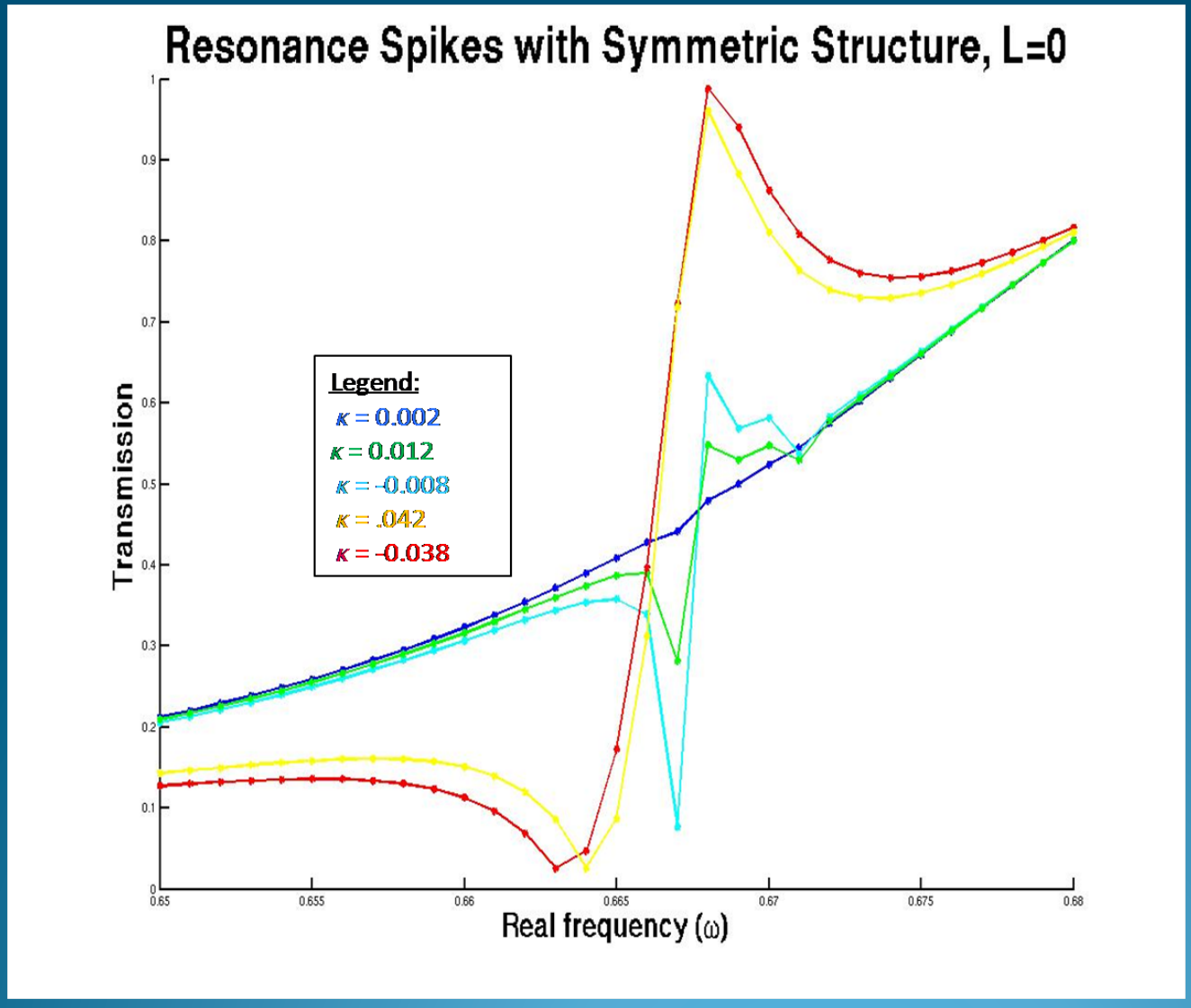
One Period of Structure



Graphical User Interface (GUI)

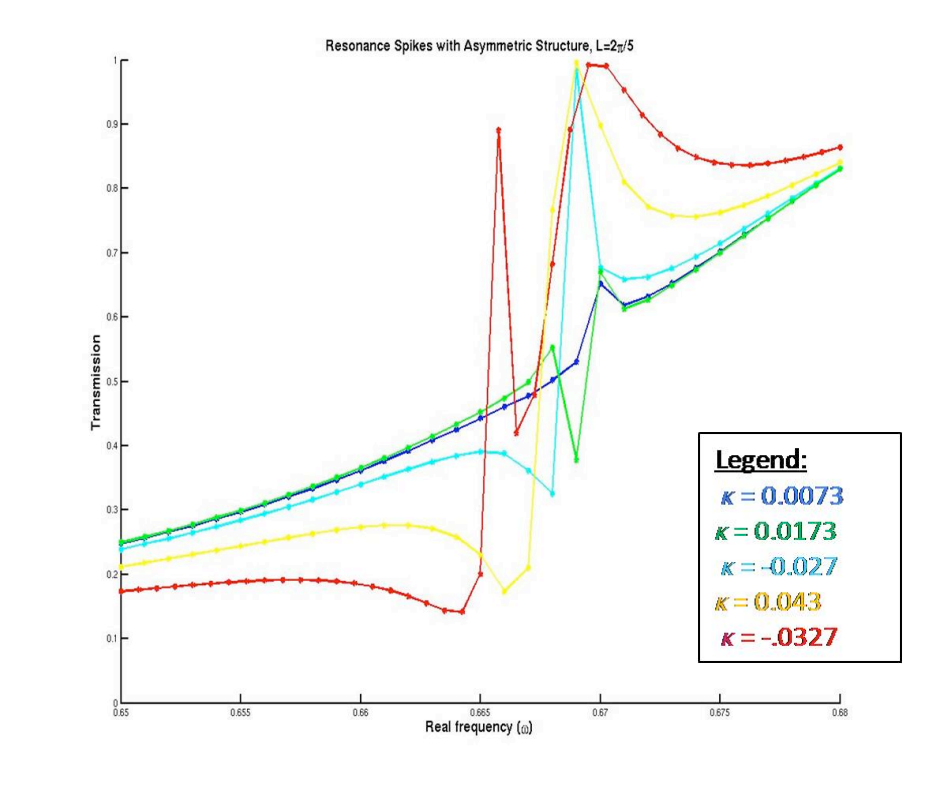


Results



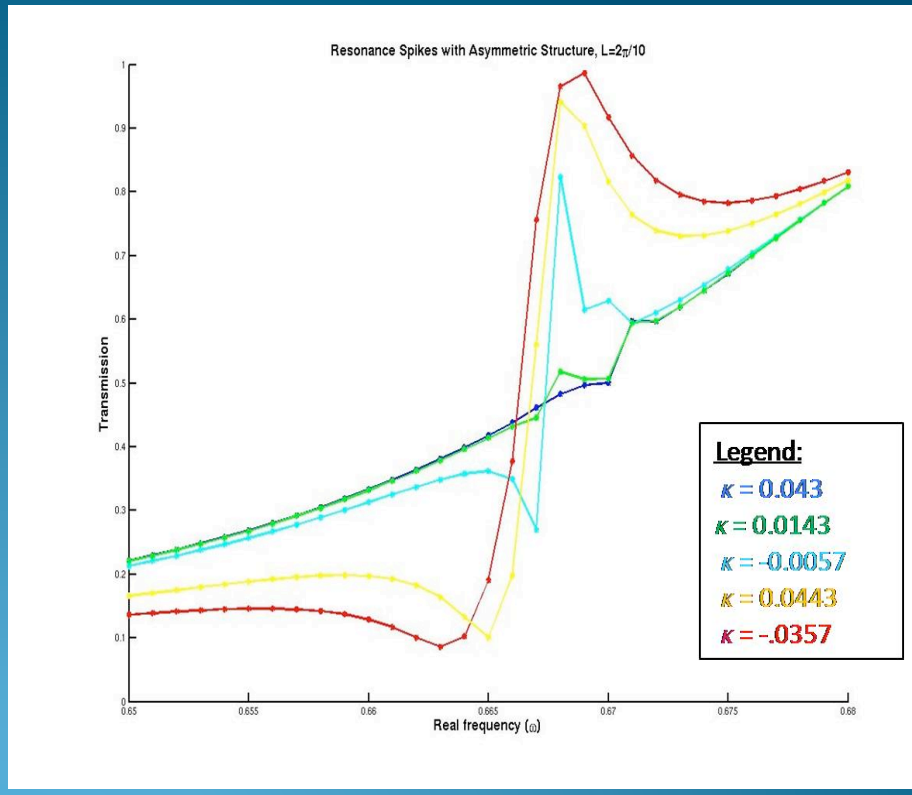
The spike emanates from the parameters κ=.002 and ω=. 66542 of a guided mode.

Results

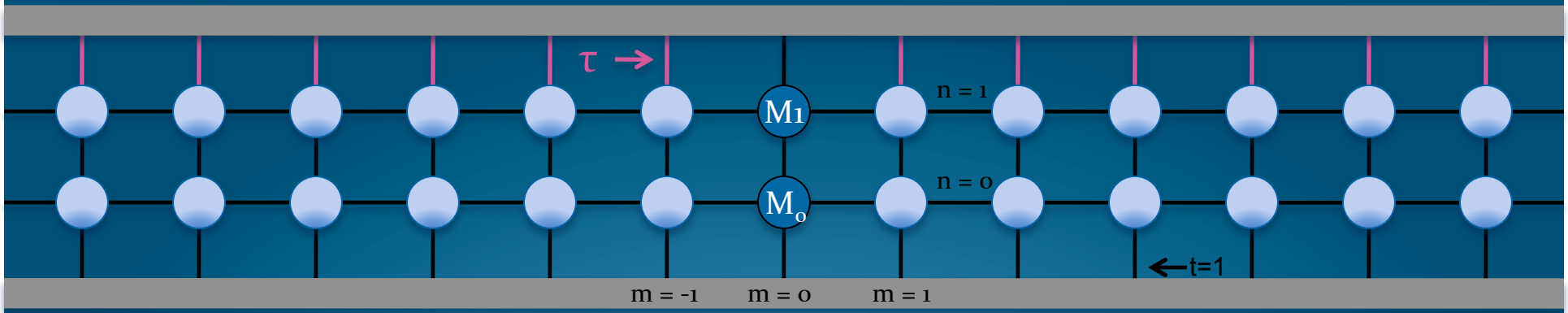


The spike emanates from κ=.0073 and ω=.6653.

The spike emanates from κ=.043 and ω=.66517.

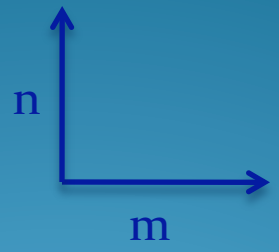


Discrete Structure



Variables:

ω: frequency
τ: tension, biases the ambient lattice
M_o : structural parameter



Discrete Structure

Newton's Second Law:

$$M_{mn}\ddot{U}_{mn}(t) = U_{m\tilde{n}}(t) + U_{m-1,n}(t) + U_{m+1,n}(t) - (\tau + 3)U_{mn}(t)$$

Time-Harmonic Solutions:

$$U_{mn}(t) = u_{mn}e^{-i\omega t}$$

Time Independent Balance of Forces Equation:

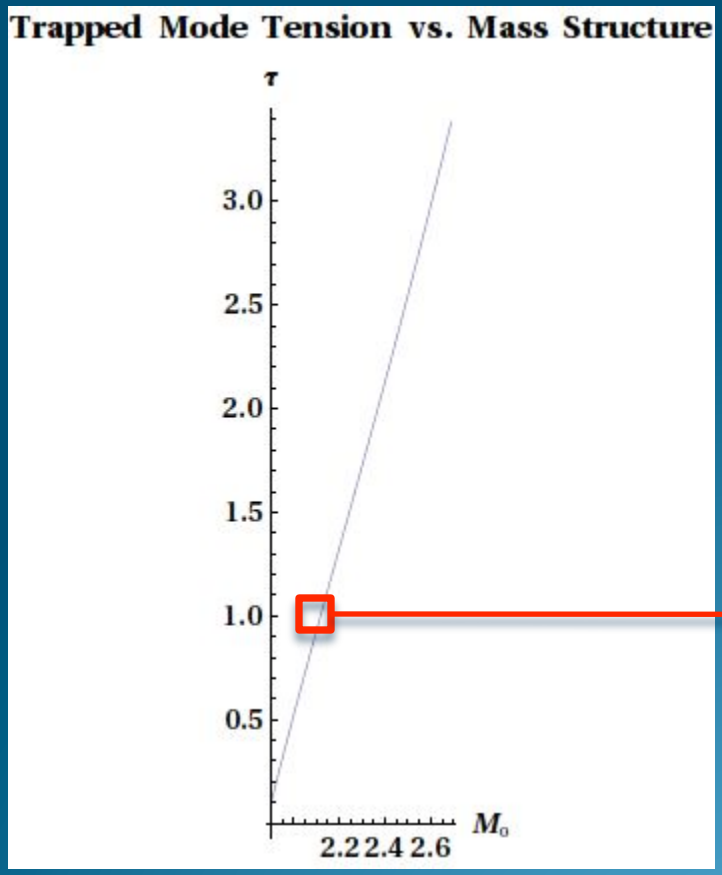
$$0 = u_{m\tilde{n}} + u_{m-1,n} + u_{m+1,n} + (M_{mn}\omega^2 - \tau - 3)u_{mn}$$

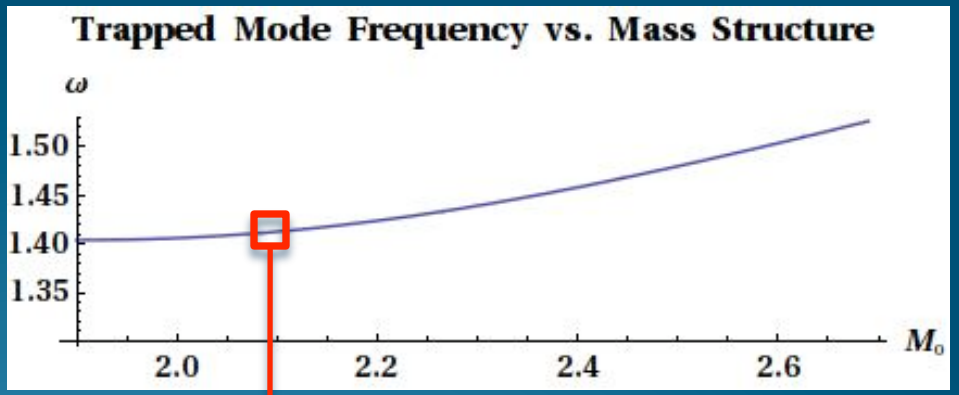
General Solution to the Scattering Problem:

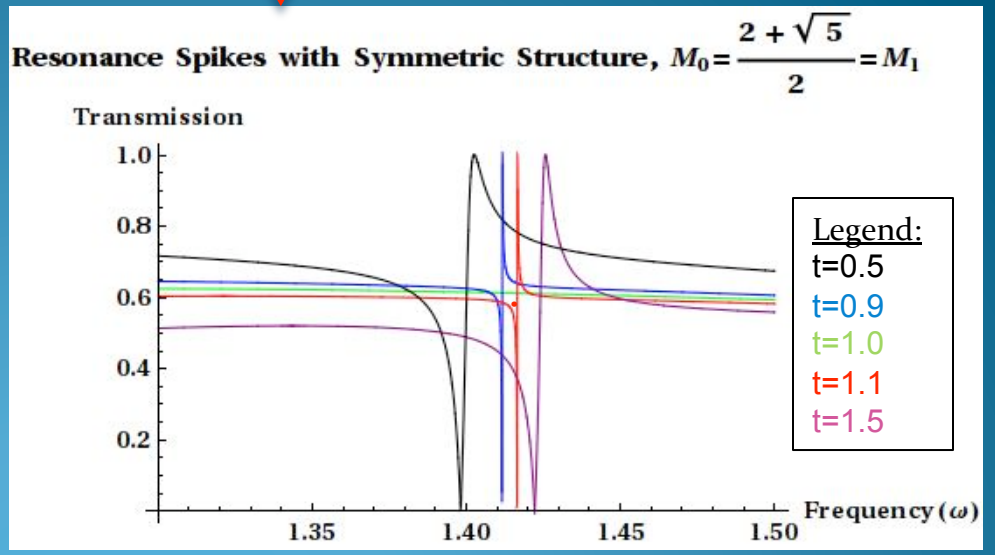
$$u_{mn} = \begin{cases} q_n^p e^{i\theta m} + (A^p q_n^p e^{-i\theta m} + A^e q_n^e e^{\alpha m}) & (m < 0) \\ (B^p q_n^p e^{i\theta m} + B^e q_n^e e^{-\alpha m}) & (m > 0) \end{cases}$$

Transmitted Energy: $T^2 = |B^p|^2$

Trapped Modes

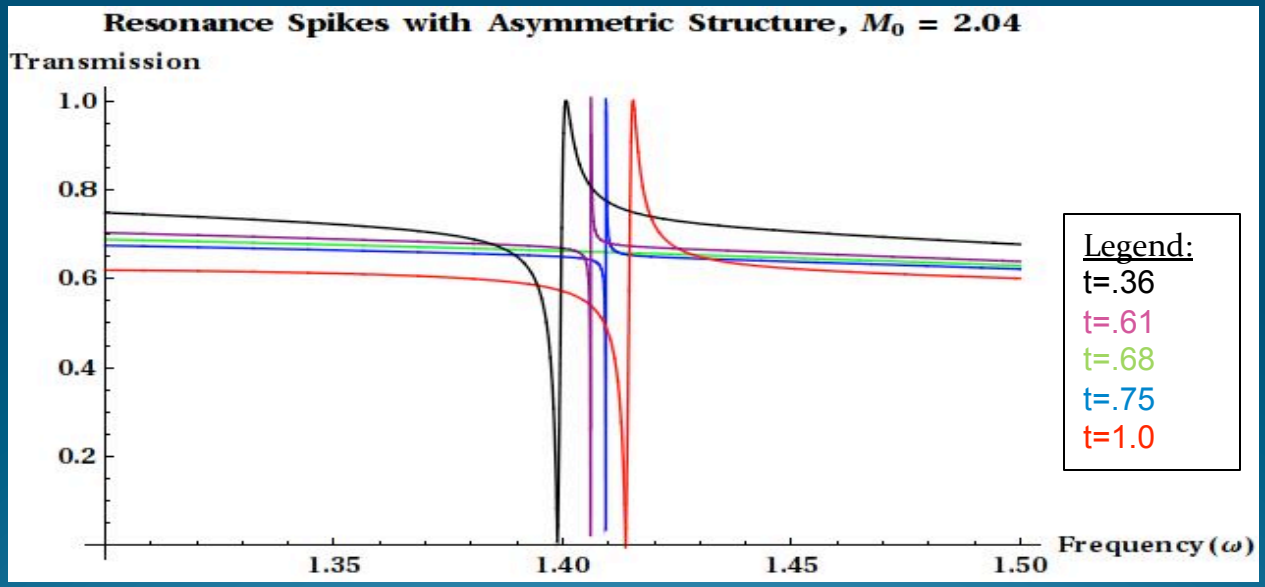


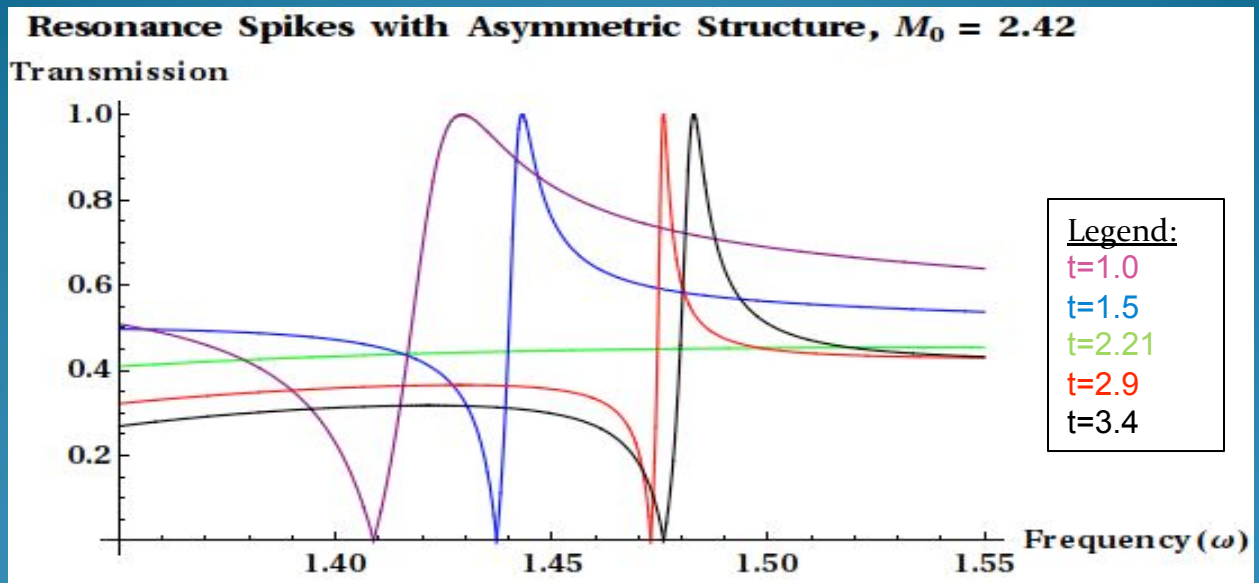




Symmetric Structure $M_0=M_1=\frac{2+\sqrt{5}}{2}$
admits a trapped mode for:
t=1 w= $\sqrt{2}$

Transmission





Future Work

Future works will be directed towards finding a universal principle that governs transmission anomalies when a structure is generically perturbed.