

Math 7384 : Problems

1. Prove that the integrator functions σ in Theorem 1 and in Theorem 2 of the class notes are of bounded variation on $[0, 2\pi]$.

2. Find α , μ and $\sigma(z)$ so that the integral formula of Theorem 2, p. 10, produces the following functions defined on the upper half plane:

$$(a) f(z) = -\frac{1}{z}$$

$$(b) f(z) = \frac{4z}{1-z^2}$$

$$(c) f(z) = i$$

3. Find an integral representation akin to those of Theorems 1, 2, 3 that characterizes exactly those analytic functions defined outside the closed unit disk whose real part is nonnegative.

$|z| > 1$

Prove that your representation holds if and only if the function has these properties (analytic and $\operatorname{Re} f \geq 0$).