Math 7384 Problem Set 2

4. Let $\{E_t\}_{t\in\mathbb{R}}$ be a resolution of the identity for a self-adjoint operator in a Hilbert space.
   * Prove that $E_s \to E_t$ strongly as $s \to t^-$, that is, the family is left continuous. [Recall that, by convention, we take the spectral functions $\sigma(t;u)$ to be left continuous.]
   * Prove that there is a resolution of the identity that is not left continuous in the operator (or uniform) norm [again assuming left continuity of $(E_t u, v)$].

5. If T is a self-adjoint operator in $\mathcal{H}$ with spectral resolution $\{E_t\}_{t\in\mathbb{R}}$, prove that E_t reduces $R_z = (T-z)^{-1}$, that is,

$$E_t R_z = R_z E_t \quad \forall\ t \in \mathbb{R}\ \ \forall\ z \in \mathbb{C}\setminus\mathbb{R}.$$

6. Express the resolvent of $-i\partial$ on $\mathbb{R}$ by means of a convolution integral. Do this for each $z \in \mathbb{C}\setminus\mathbb{R}$ and pay attention to the role of the sign of $\operatorname{Im} z$.
   One can do this using ODE theory, but I want you to do it using the spectral resolution of the resolvent $(-i\partial - z)^{-1}$.