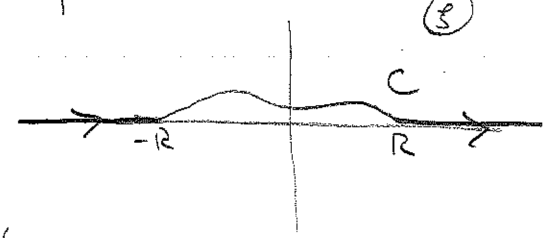
2012 F Math 7386 Problem Set 4

12. Consider solutions of the equation $u''(x) + \mu u(x) = f(x)$, with $f \in C_c^\infty(\mathbb{R})$ defined through the formula

$$u(x) = \int_C \frac{\hat{f}(\xi)}{P(\xi)} e^{2\pi i \xi x}\, d\xi ,$$



where C is a contour that includes the intervals $(-\infty, -R)$ and (R, ∞) and does not contain any roots of the symbol P of the operator $L = \partial_x^2 + \mu$.

There are four distinguished solutions, depending on where the roots of $P(\xi)$ fall relative to C, which are characterized by their behavior for $x \to \infty$ and $x \to -\infty$.

Derive the large $|x|$ behavior of each of these solutions for general $\mu \neq 0$ as well as for $\mu = 0$.

13. Extend this analysis to linear ODEs $Lu = f$, with $f \in C_c^\infty(\mathbb{R})$ and $L = \sum_{\alpha=1}^{k} a_\alpha \partial_x^\alpha$.