The integer cohomology algebra of toric arrangements

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joint work with Emanuele Delucchi (Univ. of Fribourg, CH)

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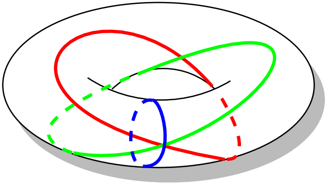

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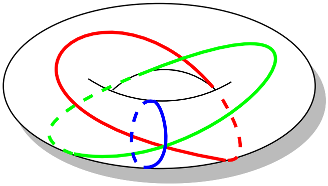

For each i,

$$K_i = \{z | \chi_i(z) = b_i\}$$

with $\chi_i(z) = z^{a_i}, a_i \in \mathbb{Z}^d$, a *primitive* character (i.e. K_i's are connected). We assume $b_i \in \mathbb{C}^*$.

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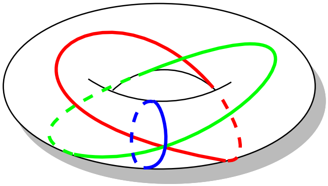

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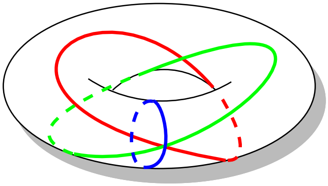

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We also assume $\mathrm{rk}[a_1, \ldots, a_n] = d$, i.e. $\mathcal{A}$ *essential* (minimal non-zero intersections have dimension 0).

Topology

The *complement* of the arrangement is

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Problem: determine the ring $H^*(M(\mathcal{A}), \mathbb{Z})$. Is it combinatorial?

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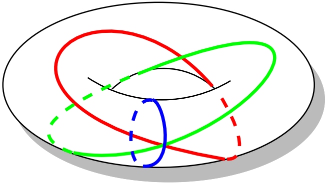
We give an answer depending on two ingredients:

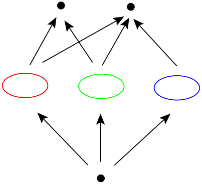
- Brieskorn decomposition for hyperplane arrangements (combinatorial);
- Maps induced by inclusion of subtori in T (depend on equations).

Combinatorics

Define the poset of layers (with rev. inclusion):

$\mathcal{C}(\mathcal{A}) := \{L \subset T \mid L$ is a c.c. of an intersection of elements of $\mathcal{A}\}$





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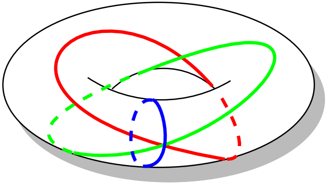

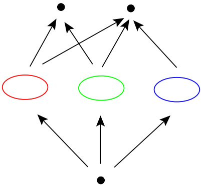

Theorem (Looijenga '95, De Concini-Procesi '05)

$$\mathrm{Point}(M(\mathcal{A}), \mathbb{Q}) = \sum_{L \in \mathcal{C}(\mathcal{A})} \underbrace{\mu_{\mathcal{C}(\mathcal{A})}(\hat{0}, L)}_{\text{Möbius function of } \mathcal{C}(\mathcal{A})} (-t)^{\mathrm{rk}\, Y} (1-t)^{d - \mathrm{rk}\, Y}$$

Möbius function of $\mathcal{C}(\mathcal{A})$

Real complexified arrangements

We call $\mathcal{A}$ *real complexified* if $\mathcal{A} = \{\chi_i^{-1}(b_i)\}$ with $b_i \in S^1$. It induces a polyhedral cellularization of $(S^1)^d$.

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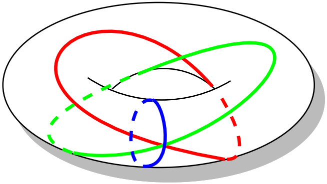

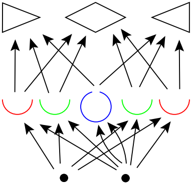

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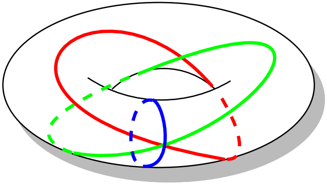
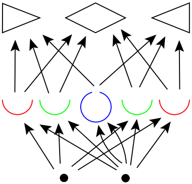




Theorem (d'Antonio-Delucchi '12)

The data of $\mathcal{F}(\mathcal{A})$ determines an CW-complex $\mathrm{Sal}(\mathcal{A})$ such that

$$\mathrm{Sal}(\mathcal{A}) \simeq M(\mathcal{A})$$

Known results

[De Concini-Procesi '05] Computation of the cup product in $H^*(M(\mathcal{A}), \mathbb{C})$ and formality when the matrix $[a_1, \ldots, a_n]$ is unimodular.

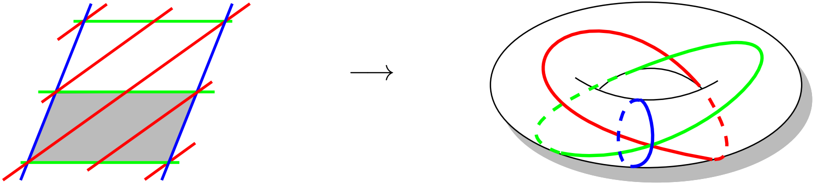
[Bibby '14] Rational cohomology algebra for unimodular abelian arrangements.

[Delucchi-d'Antonio '13] For real complexified toric arr's: minimality of $M(\mathcal{A})$ and hence $H^*(M(\mathcal{A}), \mathbb{Z})$ is torsion free.

[Dupont '15] Complements of hypersurface arrangements (in particular toric arrangement) are formal.

Coverings

Via the universal cover $\mathbb{C}^d \xrightarrow{\pi} T$ the toric arrangements $\mathcal{A}$ lift to an infinite periodic hyperplane arrangement $\mathcal{A}^{\upharpoonleft}$.



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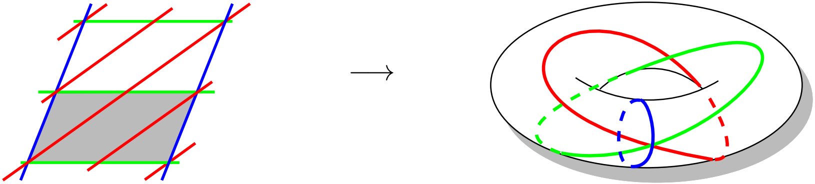
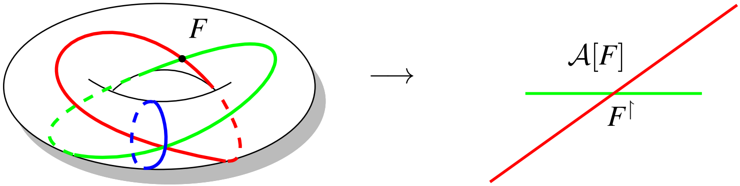


For a layer $L \in \mathcal{C}(\mathcal{A})$ we can choose a lifting $L^{\upharpoonright}$ in the $\mathcal{C}(\mathcal{A}^{\upharpoonright})$. We define $\mathcal{A}[L]$ as the central hyperplane arrangement $\mathcal{A}^{\upharpoonright}_{L}$.



Toric Salvetti complex, I

Theorem (C.-Delucchi '15)

For every real complexified toric arrangement $\mathcal{A}$*, given a layer* $L \in \mathcal{C}(\mathcal{A})$ *there is a CW-complex* $S_L \subset \mathrm{Sal}(\mathcal{A})$ *that is homotopy equivalent to* $L \times \mathrm{Sal}(\mathcal{A}[L])$*.*

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Example: $\mathcal{A} = \{1\} \subset T = \mathbb{C}^*$

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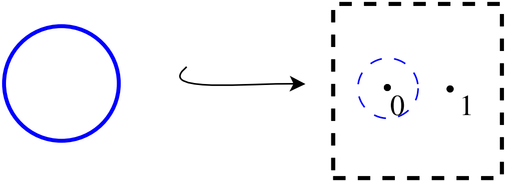

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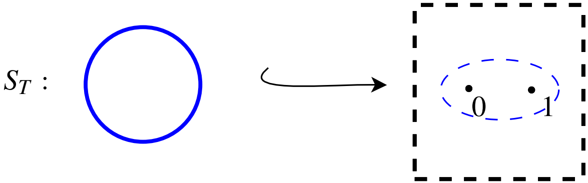

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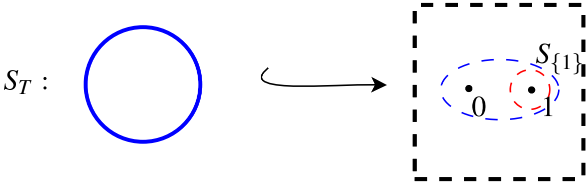

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The projection/inclusion map to/into the torus gives a commutative diagram

$$\begin{array}{ccc} \mathrm{Sal}(\mathcal{A}) & \xrightarrow{\simeq} & M(\mathcal{A}) \\ \downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle i} \\ (S^1)^d = T_c & \xrightarrow{\simeq} & T \end{array}$$

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The vertical maps induce a Leray cohomology spectral sequence (see [Bibby '14])

$$E_2^{p,q} = \bigoplus_{\substack{L \in \mathcal{C}(A) \\ \mathrm{rk}\, L = q}} H^p(L) \otimes H^q(M(\mathcal{A}[L]))$$

Toric Salvetti complex, II

For every layer $L \in \mathcal{C}(\mathcal{A})$ we have a commuting diagram:

$$\begin{array}{ccc} L \times \mathrm{Sal}(\mathcal{A}[L]) \simeq S_L & \xrightarrow{\subset} & \mathrm{Sal}(\mathcal{A}) \simeq M(\mathcal{A}) \\ \downarrow \pi_L & & \downarrow \pi \\ L \cap T_c = L_c & \xrightarrow{\subset} & T_c \end{array}$$

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where

$${}_LE_2^{p,q} = H^p(L) \otimes H^q(M(\mathcal{A}[L]))$$

(all cohomologies with $\mathbb{Z}$-coefficients)

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Hence we have:

$$\begin{array}{ccc} H^*(M(\mathcal{A})) & \longrightarrow & H^*(L) \otimes H^*(M(\mathcal{A}[L])) \\ \simeq \downarrow & & \simeq \downarrow \\ E_2^{p,q} & \longrightarrow & {}_L E_2^{p,q} \end{array}$$

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We will examine the morphism of spectral sequences associated to the map

$$\sqcup_{L \in \mathcal{C}(\mathcal{A})} S_L \to \mathrm{Sal}(\mathcal{A}).$$

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Since realizable arithmetic matroids containing a unimodular base have an essentially unique realization:

Corollary

If $[a_1, \ldots, a_n]$ contains an unimodular base, then $\mathcal{C}(\mathcal{A})$ determines the cohomology ring.

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From the surjection (induced by $M(\mathcal{A}) \hookrightarrow \prod_P M(\mathcal{A}_P)$)

$$\bigotimes_{\substack{P \in \mathcal{C}(\mathcal{A}) \\ P \text{ point}}} H^*(M(\mathcal{A}_P)) \to H^*(M(\mathcal{A}))$$

we get a complete description of the ring $H^*(M(\mathcal{A}))$.

Main Result

Theorem (C.-Delucchi)

Let $\mathcal{A}$ be any toric arrangement. The ring $H^*(M(\mathcal{A}))$ is isomorphic to the image of ϕ:

$$\phi : \bigoplus_{\substack{L' \in \mathcal{C}(A) \\ \operatorname{rk} L' = q}} H^p(L') \otimes H^q(M(\mathcal{A}[L'])) \to \bigoplus_{L \in \mathcal{C}(\mathcal{A})} H^*(L) \otimes H^*(M(\mathcal{A}[L]))$$

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given on the L'-summand by:

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Questions and remarks

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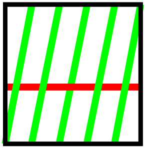

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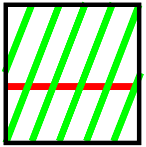

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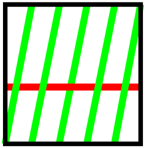

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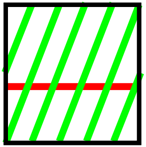

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Happy birthday Mike!