Rock Physics and Depositional History from Seismic Matching: A model study

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Introduction

In this study, we apply advanced numerical optimisation techniques to extend the work of Imhof and Sharma (2005, 2006) to integrate geological and geophysical data and infer the sedimentary parameters that produce a match to seismic data. In particular, we seek to match not just event timing (phase) but also reflection strength (amplitude). This inverse problem of quantitatively matching present-day measurements of structure, stratigraphy, petrology and/or fluids is inherently ill-posed and computationally difficult. In our approach we automatically adjust parameters, which control numerical forward models such as numerical basin models, petrophysical models, and seismic acoustic models, to match observed seismic data and observed stratigraphy.

The Study

To design our proof of concept study we selected Basin2 Release 5.0 (Bethke, Lee and Park, 2002) as our forward basin modelling package. This mature 2D process-driven numerical hydrogeological flow and transport program has been developed and improved for more than a decade with the input and support of 20 industrial, government and academic sponsors and provides the ability to specify physical and hydrologic sediment properties, e.g. sand and shale content, porosity, permeability, fluid pressures and temperature history. In addition it is freely available, making it eminently suitable for comparisons and intercompany collaboration. Moreover the needed models calculations are fast enough that the hundreds or thousands of iterations that we expected our inversion/optimisation algorithms to invoke are feasible.

Having selected a suitable sedimentary simulator, we next constructed a model. Here again the aim was to have it sufficiently realistic that it would expose many of the challenges in successfully applying state-of-the-art optimisation, while simultaneously not be so large that progress in understanding and overcoming issues would slow to a crawl. We opted for a progradational environment, with sand and shale sediment supplied from the more proximal right-hand-side of the model, and we used Basin2 to forward evolve the basin as shown in Fig. 1. The parameter space has two lithologies, yielding 6 Basin2 inputs to define porosity, and 10 layers, with sand-shale content determined by two Gaussian parameters (μ and σ) per layer, yielding a total of 26 parameters for this model.

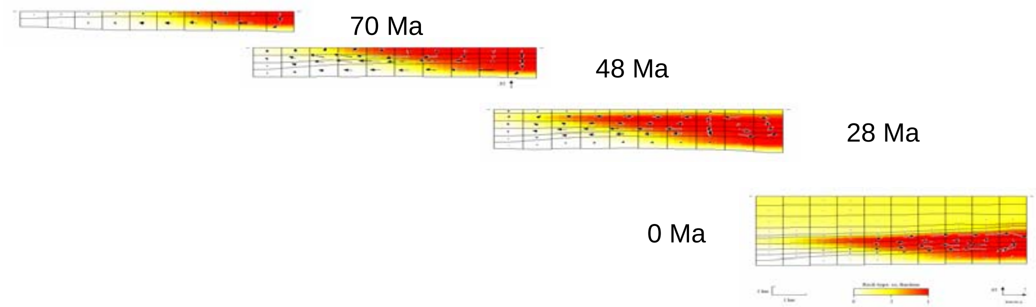


Figure 1. *Basin2 evolution of our sedimentary model. Note that the more proximal right-hand side of the model has generally higher sand-to-shale ratios.*

Now that we had our "ground truth" geologic and lithologic model, the final step was to construct seismic data as input to our study. Fig. 2 displays the key variables and the empirical rock physics relationships used to generate the initial seismic reflectivity and data.

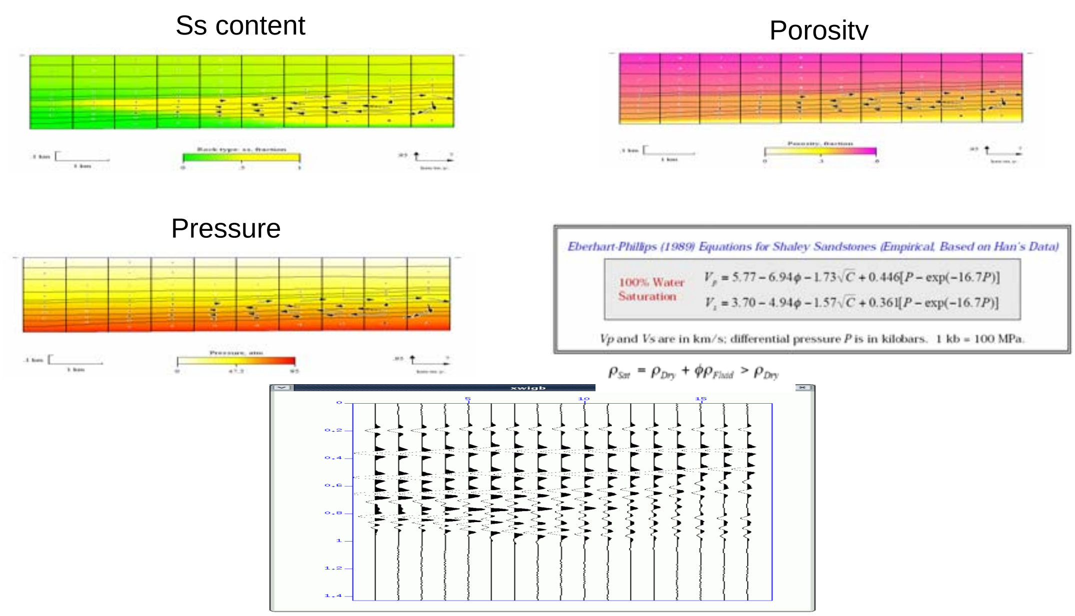


Figure 2. *Key variables and relations used for our seismic modelling of the "true" data at the bottom.*

To tackle parameter inversion/optimization for this model, we experimented with a number of algorithms, some in the derivative-free (DF) class and the remainder requiring an analytical or a numerical derivative (gradient-based). In the derivative-free category are: (1) DFO (Derivative Free Optimization) from http://projects.coin-or.org/Dfo/ due to Conn, et al. (1997, 1998); (2) NEWUOA algorithm (Michael J.D. Powell, 2004), and (3) ASA (Adaptive Simulated Annealing) from http://www.ingber.com/#ASA developed by Ingber (1989). In the gradient-based methods are: (1) IPOPT (Interior Point OPTimizer) from http://projects.coin-or.org/Ipopt/ which implements and extends the work of Wächter (2002); (2) CBB (Cyclic Barzilai-Borwein) from Dai, et al. (2006) and a Conjugate Gradient CGCBB variant (Hager & Zhang, 2006), and (3) LMDIF (Levenberg-Marquardt w/forward DIFerence Jacobian estimation) from the MINPACK library at http://www.netlib.org/minpack/ due to Moré (1978).

Unlike traditional economic applications of optimization, where one optimizes, say, net present value dollars of return, seismic matching has the additional challenge of constructing a “good” measure of fit. In the initial stages of our present investigation, we encountered poor convergence, or even nonconvergence, in directly applying least-squares optimization. As exemplified in Fig. 3, this traditional sum of squared differences between the actual and modeled seismic amplitude data exhibited many (spurious) local minima. This told us we needed a smoother measure of seismic match. For the related challenge of velocity inversion of prestack data, the differential semblance optimization (DSO) is the only provable asymptotically smooth optimization functional with the correct problem-dependent minima (Stolk and Symes, 2003) and is why the approach of Symes (1991) has proven so useful.

The problem we tackle is more complex than just estimating the velocity and reflectivity model that fits our data. To make headway we, perhaps unnecessarily, opted to separate the problem into two stages, the second being traditional seismic inversion and the first the less understood challenge of recovering burial history, i.e. Basin2 input parameters, that reproduce the inverted seismic reflection model. Fig. 4 shows the functional of the “Basin2 Loop.”

Running the various optimization methods with unit weights in the function *F* produced the convergence results of Fig 5. While not ideal, they encouraged us to refocus on the complete (i.e., Basin2 and seismic) loop. For expedience, we made several modifications to use a smooth seismic functional, the envelope of the envelope of the trace, to suppress multiple minima in this first attempt at solving the full problem. In addition, because rock physics relations are mostly empirical, we added petrophysics as part of the inversion, taking the coefficients in the Eberhard-Phillips relations of Fig. 2 as unknowns. All told, this added two parameters per layer, thus adding 20 parameters to the model.

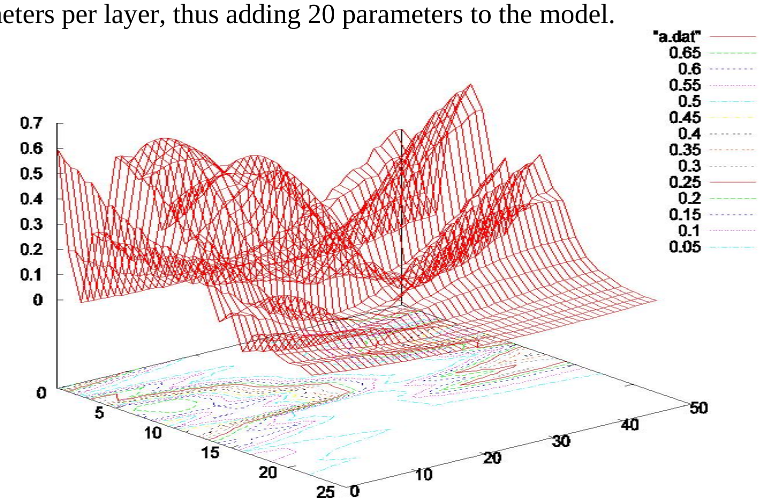


Figure 3. *Sample sum of squared differences between “true” seismic and modelled seismic data varying just two parameters of a multilayer model. Note the many local minima—an indication that the measure of fit is suboptimal.*

Basic functional

$$F = \left\| g_{sim}(\mathbf{x}) - g_{obs} \right\|_2 = w_p \left\| p_s(\mathbf{x}) - p_o \right\|_2 + w_{X_{sh}} \left\| X_{sh}^s(\mathbf{x}) - X_{sh}^o \right\|_2$$
$$+ w_\phi \left\| \phi_s(\mathbf{x}) - \phi_o \right\|_2 + w_\rho \left\| \rho_b^s(\mathbf{x}) - \rho_b^o \right\|_2$$
$$+ w_z \left\| z_s(\mathbf{x}) - z_o \right\|_2$$

Figure 4. *Basin2 Loop to fit the outputs of the Basin2 simulator to observed values. Pressure, shaliness, porosity, bulk density, and depth are matched at the center of the grid cells.*

$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^0\rvert$	initial value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^N\rvert$	final value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	number of iterations
$\approx 10^{-5}$	3.795629e–05	$< 10^{-9}$	3.363617e–18	3
$\in (10^{-4}, 10^{-3})$	8.246216e+00	$< 10^{-9}$	1.158658e–18	6
$\in (0.01, 0.2)$	7.432450e+03	$< 10^{-9}$	3.179947e–19	35
$\in (0.03, 2.9)$	1.785565e+04	$< 10^{-9}$	5.578055e–17	89

(a) LMDIF

$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^0\rvert$	initial value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^N\rvert$	final value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	number of iterations
$\approx 10^{-5}$	3.795629e–05	$\in (10^{-4}, 2.9)$	7.3350556e-03	200
$\in (10^{-4}, 10^{-3})$	8.246216e+00	$\in (10^{-5}, 10^{-3})$	3.6374664e-06	200
$\in (0.01, 0.2)$	7.432450e+03	$\in (10^{-4}, 0.2)$	4.4696880e-04	200
$\in (0.03, 2.9)$	1.785565e+04	$\in (0.002, 5.5)$	2.4107763e-01	200

(c) IPOPT

$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^0\rvert$	initial value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^N\rvert$	final value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	number of iterations
$\approx 10^{-5}$	3.795629e–05	$\in (10^{-5}, 10^{-4})$	0.1574337e-05	455
$\in (10^{-4}, 10^{-3})$	8.246216e+00	$\in (10^{-4}, 0.13)$	0.3358357	502
$\in (0.01, 0.2)$	7.432450e+03	$\in (0.03, 3.3)$	1.970609e+01	502
$\in (0.03, 2.9)$	1.785565e+04	$\in (0.02, 4.75)$	2.036483e+01	517

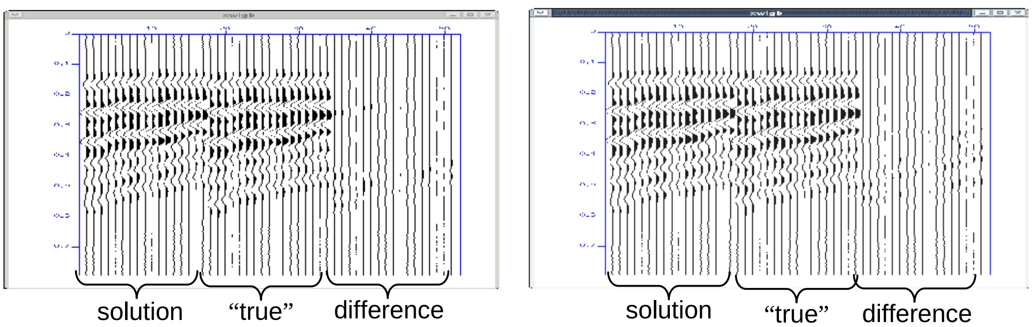
(b) DFO

$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^0\rvert$	initial value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	$\lvert\mathbf{X}_i^{true}-\mathbf{X}_i^N\rvert$	final value of $\lvert\lvert\boldsymbol{F}\rvert\rvert_2^2$	number of function evaluations
$\approx 10^{-5}$	3.795629e–05	$\in (0.0000047, 0.01)$	1.4764192e-05	223
$\in (10^{-4}, 10^{-3})$	8.246216e+00	$\in (0.0003, 1.35)$	6.598292e-02	1475
$\in (0.01, 0.2)$	7.432450e+03	$\in (0.006, 4.14)$	2.101843e-02	2736
$\in (0.03, 2.9)$	1.785565e+04	$\in (0.002, 2.11)$	6.762479e-03	2486

(d) NEWUOA

Figure 5. *Convergence behavior of selected optimization methods for Basin2 Loop. Each line from the table corresponds to a different starting guess.*

With such modifications, we achieved the very respectable seismic fit shown in Fig. 6. Both the ASA and NEWUOA methods required well over a thousand iterations in order to achieve this degree of fit.



(a) **(b)**

Figure 6. *Full Basin2→petrophysics→seismic fit using (a) ASA and (b) NEWUOA.*

Summary

There are plenty of aspects to this model study problem meriting further attention. We still want to adapt to the differential semblance measure of Symes to smooth the functional being optimised. We continue to explore scaling and other preconditioning to improve convergence speed and conserve computational resources. And, ultimately, we must stochastically explore the space of parameters that provide acceptable data matches, not only to the seismic, but also to other measurements such as well logs. Based upon our progress to date, however, we are reasonably confident that our basic approach and machinery are feasible with modern industrial computing resources for tackling these larger problems of incorporating field data.

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