

Symmetric drawings of the Hoffman-Singleton graph

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For a description of the Hoffman-Singleton graph, and several references, see [2]. There you will find “A beautiful symmetric embedding due to E. Pegg Jr.”, whose page on this graph is [1]. (This is actually a drawing rather than an embedding; the graph is not planar.) The drawing obviously has a rotational symmetry of order 5. Less obviously to the naked eye, it has no reflection symmetry, so the symmetry group is C_5 . It also has the property that vertices that are adjacent on the circle are adjacent in the graph. I did a computer search for all circular drawings with this last property and with at least five-fold rotational symmetry. There are 666 of them (up to the action of the dihedral group of order 100), and they all have symmetry group C_5 . Here are a few; the numbering is assigned by the order in which the program spit them out. The drawing in Figure 1 is Ed Pegg’s, and those in Figures 2–8 are more-or-less randomly selected. The drawing in Figure 9 is the unique one with the minimum number of different chord lengths (7) and that in Figure 10 is the unique one with 8 different chord lengths.

Ed Pegg’s picture shows an edge 7-coloring of the graph discovered by Gordon Royle, which is invariant under the symmetries, and also 2-colors the edges around the circle. An invariant coloring is just a coloring of the quotient graph. This graph may have loops, in which case it cannot be colored. The only ones for which the quotient graph has loops are those in Figures 9 and 10. All the rest have invariant colorings that 2-color the rim; the colorings shown in the figures were chosen by an algorithm intended to produce clusters of same-colored edges. I *think* the coloring in Figure 1 is the same as Ed Pegg’s.

References

- [1] Ed Pegg Jr., *Math Games: The Hoffman-Singleton Game*, Nov. 1, 2004.
http://maa.org/editorial/mathgames/mathgames_11_01_04.html
- [2] Eric W. Weisstein, *Hoffman-Singleton Graph*. From MathWorld—A Wolfram Web Resource.
<http://mathworld.wolfram.com/Hoffman-SingletonGraph.html>

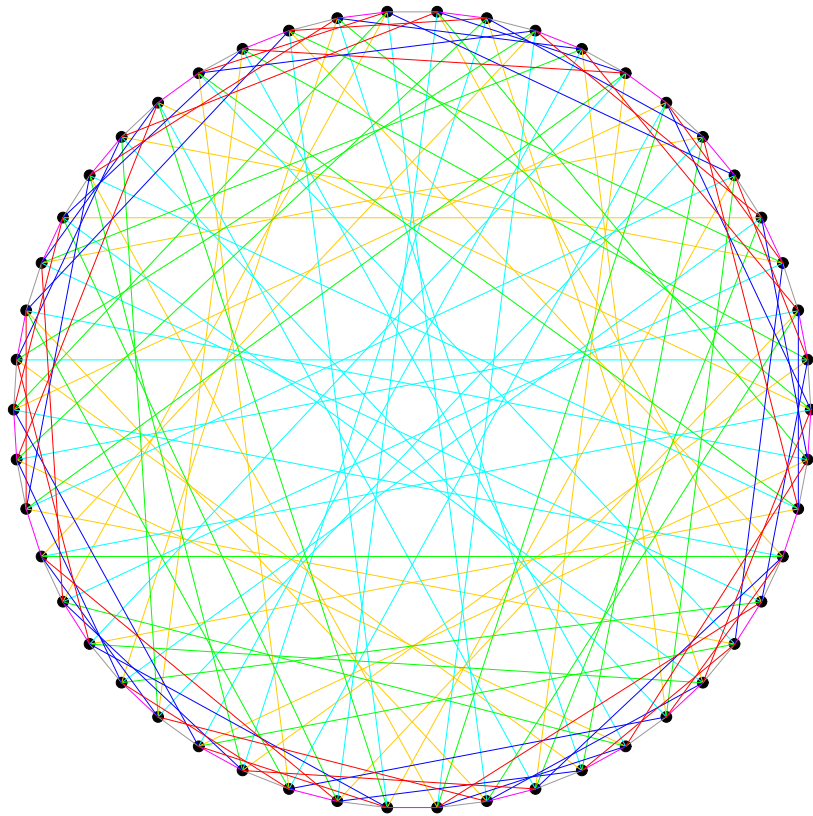


Figure 1: drawing 304

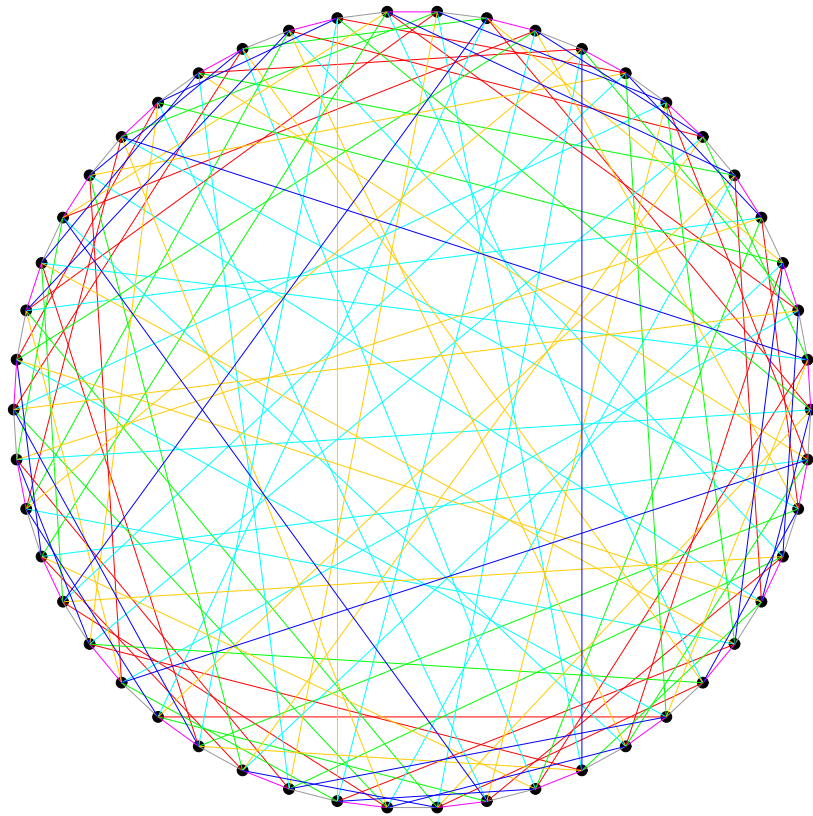


Figure 2: drawing 1

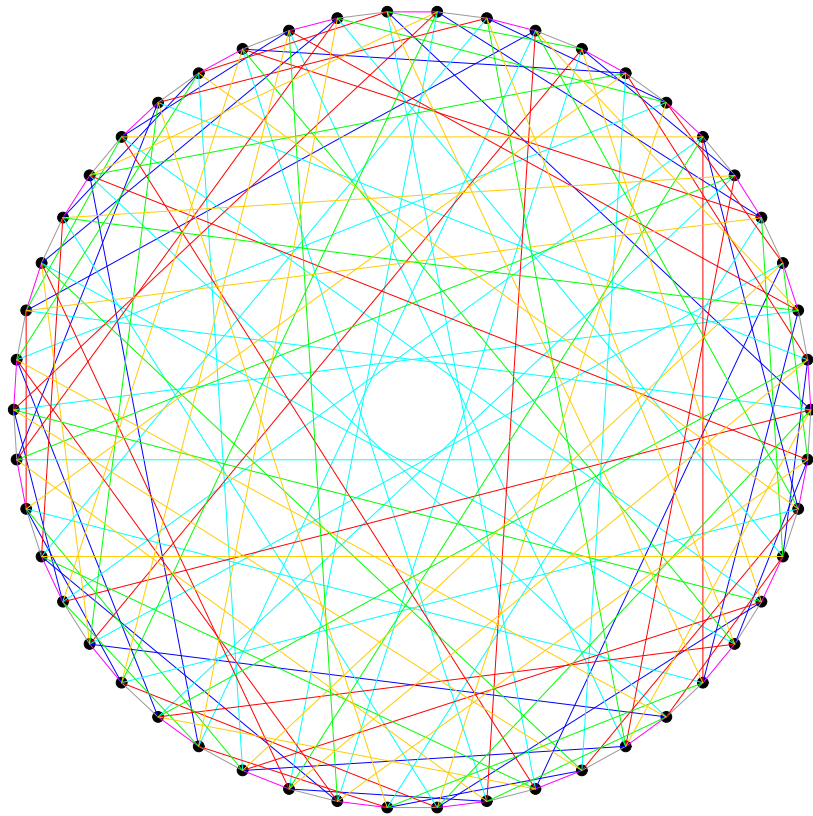


Figure 3: drawing 27

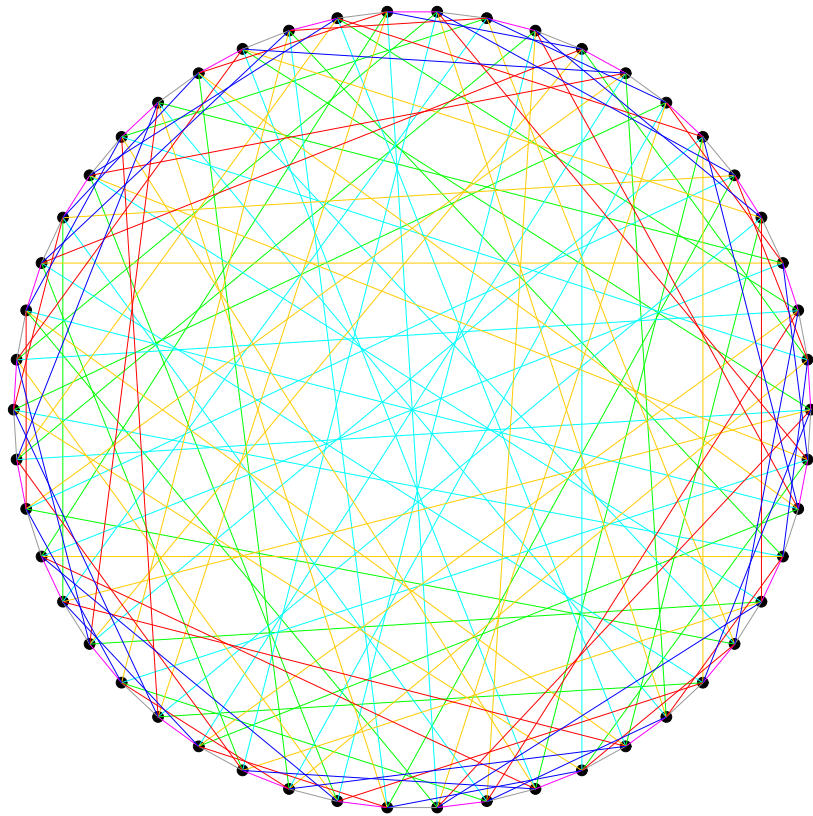


Figure 4: drawing 171

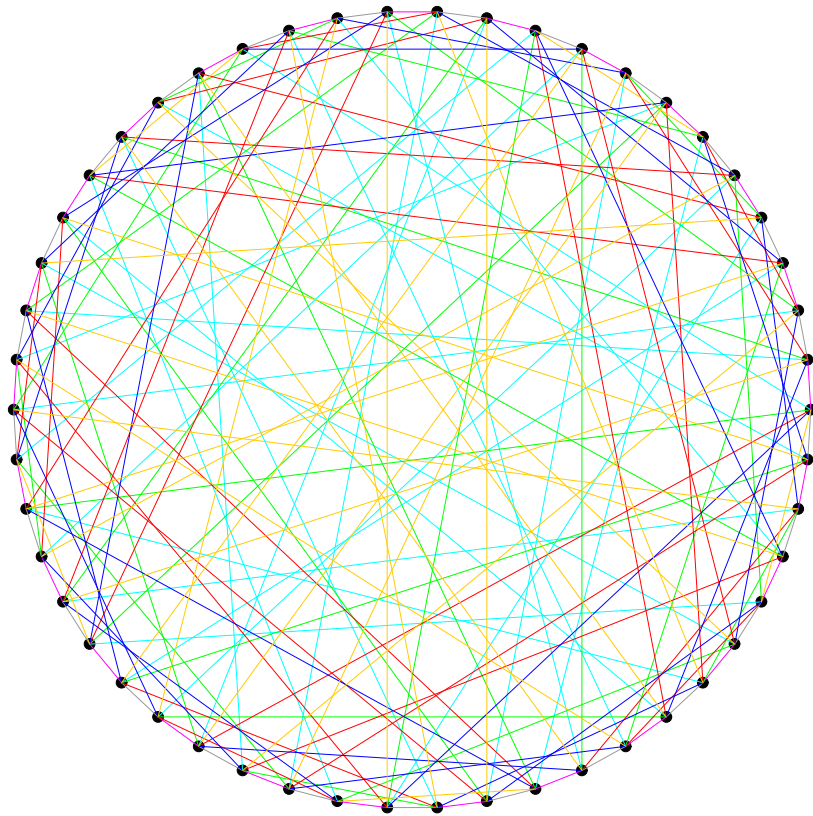


Figure 5: drawing 177

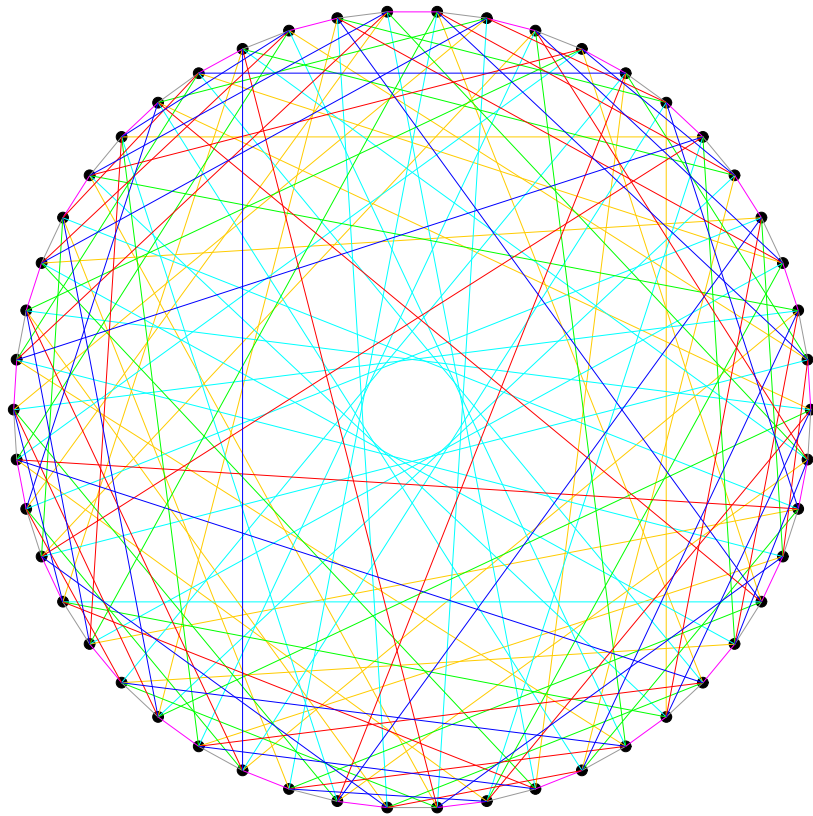


Figure 6: drawing 274

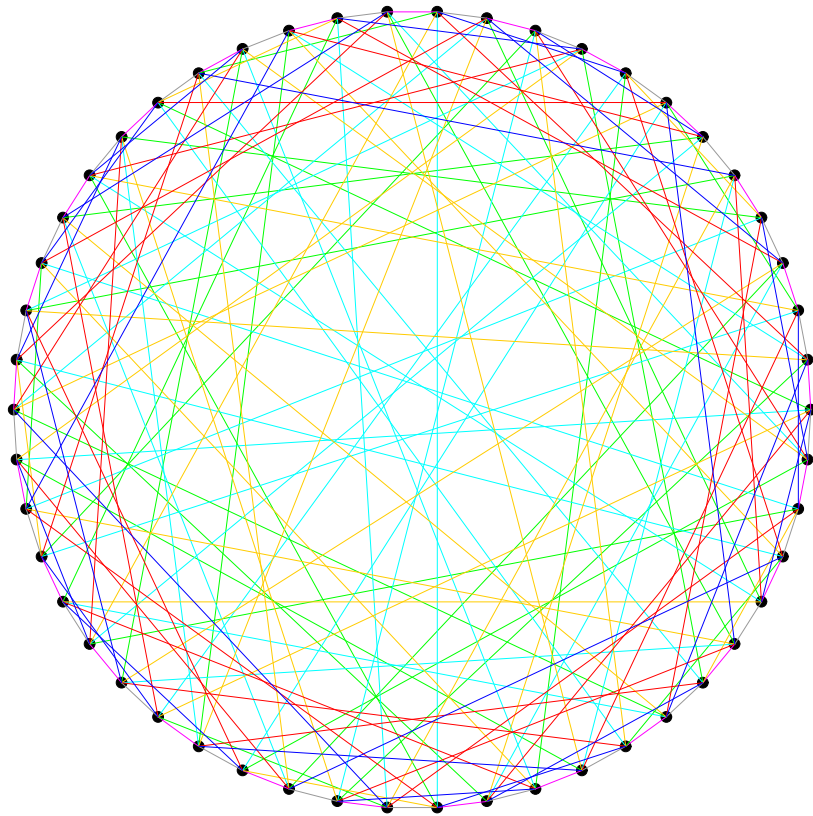


Figure 7: drawing 294

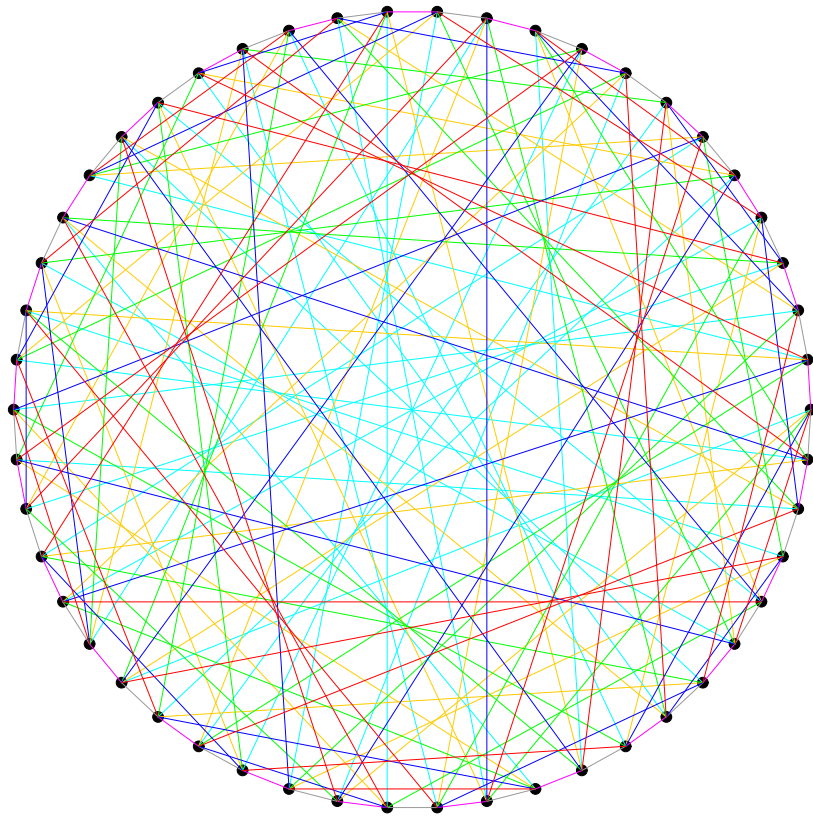


Figure 8: drawing 666

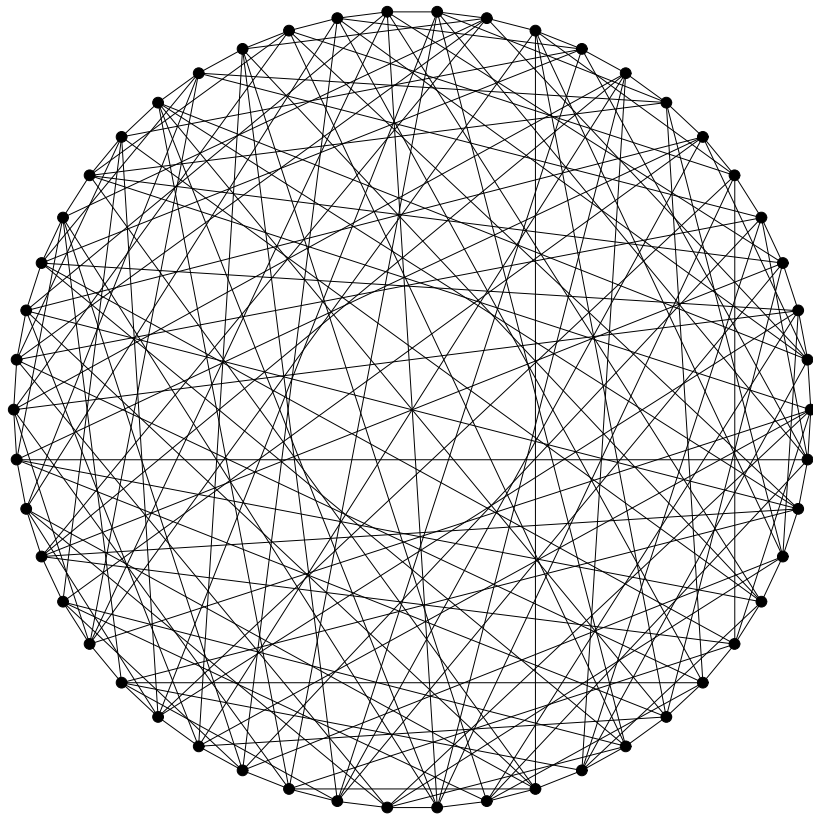


Figure 9: drawing 401

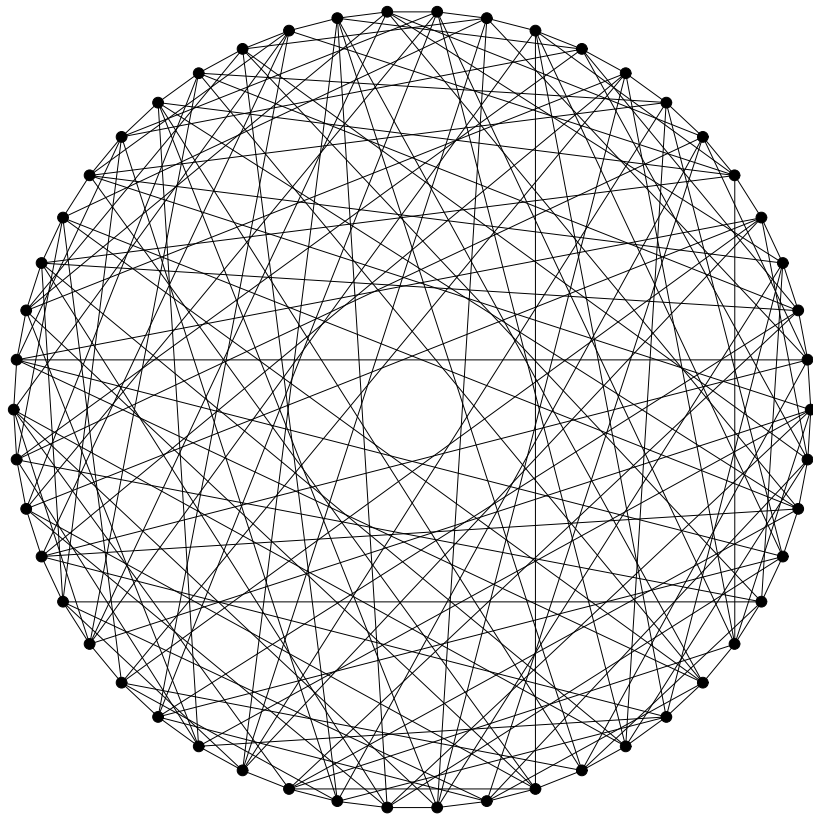


Figure 10: drawing 402