

**I. Differentiation Problems**

For each of the following functions, find the derivative with respect to  $x$ :

A. Brief problems, some of which might catch you if you're not alert. There will be 4 like this, worth 5 points each.

$x^\pi$	$e^{2x}$	$x^{2e}$	$x^e e^x$
$\sin 2x$	$\sin 2x \cos 3x$	$\frac{\sin x}{x^2}$	$\frac{\sin x}{\sin(x^2)}$
$\sin^{-1} x$	$(\sin x)^{-1}$	$\sin(x^{-1})$	$\sin^{-1}(\sin x)$

B. Problems requiring the use of the chain rule as well as other rules. There will be 3 like this, worth 10 points each.

$\tan(2x^9) + 5$	$\tan(2x^9 + 5)$	$\tan((2x + 5)^9)$	$\tan^9(2x + 5)$
$\tan(2x + 3) \sin(4x)$	$\tan(2x + 3 \sin(4x))$	$\tan((2x + 3) \sin(4x))$	$\tan(2x) + 3 \sin(4x)$
$\ln(2x + 3) \ln(4x)$	$\ln(2x + 3 \ln(4x))$	$\ln((2x + 3) \ln(4x))$	$\ln(2x) + 3 \ln(4x)$
$e^{(x^2+1)}$	$e^{(x+e^x)}$	$e^{\sin(e^x)}$	$e^{e^{e^x}}$
$\tan^{-1}(e^x)$	$(\tan(e^x))^{-1}$	$\tan(e^{-x})$	$\tan(\tan^{-1} e^x)$

C. Find the second and third derivative a function similar to one of those in part A (10 points).

**II. Implicit Differentiation**

A. Given an equation similar to any of the following, find  $dy/dx$  (10 points):

(a) $x^2 + y^2 = 1$	(b) $y^2 = x^3 - x + 1$	(c) $y^4 + 3xy^3 - 11x^3y + 8x^4 = 107$
(d) $xy^2 = (y + 1)^2 \sin(xy)$	(e) $(2 + y^2) \ln x = (x^2 + 1) \ln y$	(f) $x^2 + y^2 = ye^x$

B. Given a point on any of the curves described by such an equation, find the equation of the tangent line at that point (10 points).

For example, the point  $(2, 3)$  is on the graph of (c). *Problem.* Find the equation of the tangent line at  $(2, 3)$ .

*Solution.* Differentiating implicitly, we get

$$4y^3 \frac{dy}{dx} + 3y^3 + 9xy^2 \frac{dy}{dx} - 33x^2y - 11x^3 \frac{dy}{dx} + 32x^3 = 0.$$

When  $x = 2$  and  $y = 3$ , this becomes

$$4 \cdot 27 \frac{dy}{dx} \Big|_{(2,3)} + 3 \cdot 27 + 9 \cdot 2 \cdot 9 \frac{dy}{dx} \Big|_{(2,3)} - 33 \cdot 4 \cdot 3 - 11 \cdot 8 \frac{dy}{dx} \Big|_{(2,3)} + 32 \cdot 8 = 0$$

$$\frac{dy}{dx} \Big|_{(2,3)} = 59/182$$

So the tangent line has equation  $y - 3 = (59/182)(x - 2)$ .—On the test, the arithmetic will be easier.

**III. Related Rates.**

One of the problems 8–16 on page 260, or a similar problem. (20 points)