SYSTEMS AND CONTROL SEMINAR

3 PM, 2 October 2001, 381 Lockett Hall

Michael A. Malisoff
Assistant Professor of Mathematics
Louisiana State University

Lyapunov Functions and Viscosity Solutions, Part I

The theory of viscosity solutions forms the basis for much current research in mathematical control theory and optimization. Viscosity solutions provide a very useful framework in which non-differentiable functions can be viewed as solutions of differential equations in a generalized sense. The theory had its origins in the work of M. Crandall and P.-L. Lions during the early 1980's and is now recognized as one of the main techniques for studying optimization problems for fully nonlinear control systems. Viscosity solution theory gives criteria guaranteeing that the minimal cost function in deterministic optimal control is the unique solution of an appropriate PDE that satisfies suitable side conditions. Starting from uniqueness characterizations of this kind, viscosity solutions have been used to prove approximation results, including convergence of numerical schemes with error estimates and synthesis of nearly-optimal controls, asymptotic results for ergodic problems, and much more. This lecture will be the first in a series of talks on applications of viscosity solutions and optimal control to stabilization problems in systems theory. We will begin with a brief introduction to viscosity solutions theory and its interplay with the Dynamic Programming approach to optimal control. We will then review some recent results which characterize minimal cost functions for deterministic optimal control problems as unique viscosity solutions of the corresponding Hamilton-Jacobi equations that satisfy appropriate boundary conditions. Applications of these results to chattering control problems from control engineering, including Fuller's Example, and to eikonal and shape-from-shading problems from geometric optics and image processing will also be discussed. These uniqueness characterizations form the basis for recent PDE characterizations for robust Lyapunov functions and robust domains of attraction for locally asymptotically stable perturbed systems. These more recent characterizations will be the topic of a later talk in this series. This lecture is based in part on joint research by the speaker and Hector J. Sussmann.

SYSTEMS AND CONTROL SEMINAR

3:00 PM, 16 October 2001, 381 Lockett Hall

Michael A. Malisoff
Assistant Professor of Mathematics
Louisiana State University

Lyapunov Functions and Viscosity Solutions, Part II

This talk is the second in a series on methods of applying viscosity solutions and optimal control theory to stabilization problems. In the first talk, we introduced two important problems in control theory, namely, the problem of minimizing an integral criterion on an infinite horizon subject to control system dynamics, and the problem of constructing control-Lyapunov functions for uniformly locally asymptotically stable control systems. These problems were shown to be dual, in the sense that the minimum cost function (i.e., the value function) for the first problem is a viscosity solution of the so-called zero-discount Hamilton-Jacobi-Bellman equation (HJBE), while the unknown control-Lyapunov function in the second problem can be taken to be a viscosity solution of the so-called Zubov equation, which is the negative of an HJBE. We also discussed the Representation Theorem for viscosity solutions of HJBEs, which asserts that any viscosity solution of the HJBE satisfies certain analogs of Bellman's Dynamic Programming Principle. In this talk, we show how to use the Representation Theorem to prove that the value function for the first of our problems is the unique viscosity solution of the corresponding HJBE that satisfies appropriate side conditions. The novelty of our result is that it applies to a very general class of optimal control problems whose Lagrangians (i.e., instantaneous cost functions) are not necessarily uniformly bounded below by positive constants. By allowing more general Lagrangians, we can apply our result to HJBEs which are not tractable by means of the known results, including the HJBEs for Fuller's Example, degenerate eikonal equations, and shape-from-shading equations. However, the uniqueness result we present in this talk cannot be used to give PDE characterizations for the control-Lyapunov functions of the second problem, since the Zubov equation corresponds to an HJBE for an optimal control problem with a negative Lagrangian. This suggests the problem of how to extend the uniqueness result to problems with negative Lagrangians. This extension will be the subject of a later talk in this series. This lecture is based in part on joint research by the speaker and Hector J. Sussmann.

SYSTEMS AND CONTROL SEMINAR

3:00 PM, 6 November 2001, 381 Lockett Hall

Michael A. Malisoff
Assistant Professor of Mathematics
Louisiana State University

Lyapunov Functions and Viscosity Solutions, Part III

This talk is the third in a series on methods of applying viscosity solutions and optimal control theory to stabilization problems. In the first talk, we introduced two important problems in control theory, namely, the problem of minimizing an integral criterion on an infinite horizon subject to control system dynamics, and the problem of constructing control-Lyapunov functions for uniformly locally exponentially stable (ULES) control systems. In the second talk, we used dynamic programming methods and Zorn's Lemma to characterize the minimum cost function (i.e., the value function) in the first problem as the unique continuous bounded-from-below viscosity solution of the so-called zero discount Hamilton-Jacobi-Bellman equation (HJBE) that vanishes at the origin. Our proof was based on the Representation Theorem for viscosity solutions of HJBEs, which asserts that any viscosity solution of the HJBE satisfies certain analogs of Bellman's Dynamic Programming Principle. By allowing more general nonnegative instantaneous cost functions, this uniqueness characterization applies to HJBE's which are not tractable by means of the known results, including the HJBE for the Fuller Example from control engineering and degenerate eikonal and shape-from-shading equations from optical physics. In this talk, we will discuss these applications and the minimality of the hypotheses of our uniqueness theorem. We will then show how to use ideas from the proof of this theorem to show that the variable discount robust Lyapunov function in the second of our problems is the unique bounded viscosity solution of the Generalized Zubov equation that vanishes at the origin. We will use this result to give sublevel set characterizations of the robust domain of attraction under minimal hypotheses. Finally, we discuss general methods for constructing locally and globally Lipschitz maximal cost type robust Lyapunov functions for ULES systems. This lecture is based in part on joint research by the speaker and Hector J. Sussmann.