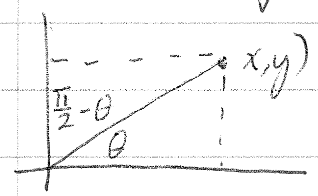
Note on Lecture of 10/7/03

We want to find $g(x,y)$ such that ① $\frac{\partial g}{\partial x} = \frac{-y}{x^2+y^2}$ and ② $\frac{\partial g}{\partial y} = \frac{x}{x^2+y^2}$

① $\Rightarrow$ (that symbol means "implies") $g(x,y) = \int \frac{-y}{x^2+y^2}\,dx = \frac{-y}{y^2}\int \frac{dx}{1+\left(\frac{x}{y}\right)^2}$

$\left[\text{letting } u = \frac{x}{y},\ du = \frac{dx}{y}\right] = -\frac{1}{y}\int \frac{y\,du}{1+u^2} = -\int \frac{du}{1+u^2}$

$= -\arctan\frac{x}{y} + c(y)$. Now let's use a correct trig identity:



$\tan\left(\frac{\pi}{2}-\theta\right) = \frac{x}{y}$, so $\frac{\pi}{2}-\theta = \arctan\frac{x}{y}$,

so $-\arctan\frac{x}{y} = \theta - \frac{\pi}{2} = \left(\arctan\frac{y}{x}\right) - \frac{\pi}{2}$

Since $g(x,y) = \arctan\frac{y}{x} + c(y) - \frac{\pi}{2}$,

$$\frac{\partial g}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} + c'(y) = \frac{x}{x^2+y^2} + c'(y)$$

$= \frac{x}{x^2+y^2}$ by ②, so $c'(y) = 0$ and

$c(y)$ is a constant.

So $\arctan\frac{y}{x}$ gives a

potential for $\vec{G}$; that is it works as g in ① and ②.

for $x>0$. We may as well say "$g(x,y) = \theta$ is a

potential for $\vec{G}$