

1. Let f be the 2π -periodic function given by

$$f(x) = |x| \quad \text{for } -\pi < x \leq \pi.$$

Then f has a Fourier series representation:

$$f(x) \approx a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Find the coefficients $a_0, a_1, b_1, a_2, b_2, \dots$. Show your procedure.

2. In each case, find the Laplace transform of the given function of t .

(1) $\sin 2t$

(2) $e^{-2t} \cos 3t$

(3) $t^2 e^t$

(4) $u_\pi(t)(t - \pi)e^{-(t-\pi)}$

(5) $\cos t$

3. In each case, find the inverse Laplace transform.

a. $\frac{1}{s-1}$

b. $\frac{1}{s^2-1}$

c. $\frac{e^{-2\pi s}}{s}$

4. Solve this initial value problem using the method of Laplace transforms, and sketch the graph of the solution:

$$y'' + 7y' + 10y = 0, \quad y(0) = 2, \quad y'(0) = -7.$$